STEP Support Programme

STEP 2 Specification Mechanics Notes

These notes are designed to help students in preparing for STEP 2. They cover the "bold and italic" sections of the STEP 2 specification which are not covered in the A-level single Mathematics specifications, or AS Further Maths Common Core. Many of these topics will be covered in A Level Further Mathematics, and will be covered in some AS Further Mathematics modules.

There are more notes on the various sections of the specification in the STEP 2 modules.

Energy, Work and Power

Energy

There are different types of energy that an object can have including **kinetic energy**, $\frac{1}{2}mv^2$, **potential energy**, mgh (where h is the height above some reference point which can usually be picked to be some convenient value), and **elastic potential energy** $\frac{\lambda x^2}{2l}$ (more on this later).

Conservation of Energy

When no external forces (apart from gravity) act (or "do work") on a particle then the total energy of the particle is unchanged. This can be extended to a system of particles.

For example, consider a particle released from a height of 2m at a speed of 3ms^{-1}. By equating the energy at the start and end you can find the speed with which the particle is moving when it hits the ground:

$$\begin{aligned} 2mg + \frac{1}{2}m \times 3^2 &= \frac{1}{2}mv^2 \\ \implies v^2 &= 4g + 9 \\ v &= \sqrt{4g+9} \end{aligned}$$

Here we have taken h to be the height above ground level, so that ground level equates to $h = 0$. This is often a sensible choice (but not always!).

Work done

The *work done* by a constant force is given by:

work done = component of force in direction of motion × distance moved

Note that if only one force acts on an object then it will move in the direction of the force and we have work done = force × distance or $W = Fd$.

The work done on a particle is equal to its change in energy. For example to move a particle up horizontally by d metres requires work of mgd which equals the gain in potential energy of the particle.

If the force varies then we have:

$$W = \int_{x_1}^{x_2} F \, \mathrm{d}x$$

where x_1 is the starting point of the particle and x_2 is the end point. If the force is given as a function of time then it might be useful to change the variable (remembering that $\frac{\mathrm{d}x}{\mathrm{d}t} = v$) to get:

$$W = \int_{t_1}^{t_2} Fv \, \mathrm{d}t$$

Power

Power is the rate at which work is done, so $P = \frac{\mathrm{d}}{\mathrm{d}t}W$. If the force is constant and acting in the direction of motion then $P = Fv$.

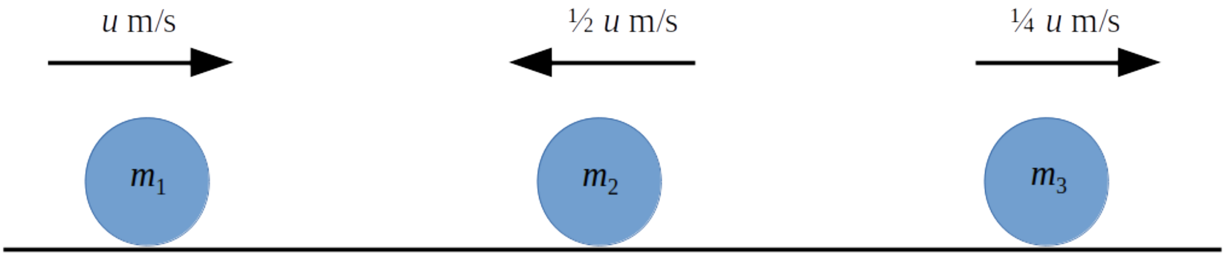
Collisions

Momentum

In systems where no external forces are applied, momentum is conserved. For example, momentum is conserved when two particles collide (as long as they are not influenced by gravity/friction etc.)

The momentum of an object is given by $m\mathbf{v}$ where $\mathbf{v}$ is the velocity of the object. It is very important to be careful with the directions that the objects are moving in — it is a good idea to draw clear diagrams of the objects before and after the collision with directions of travel indicated. I usually use positive for "moving to the right" and negative for "moving to the left".

For example, consider this situation:



in this case the total momentum of the system is $um_1 - \frac{1}{2}um_2 + \frac{1}{4}um_3$.

In general for two particles we have:

$$m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$$

where $\mathbf{u_1}, \mathbf{u_2}, \mathbf{v_1}, \mathbf{v_2}$ are all velocities "to the right" (so if one of the particles was moving in the opposite direction that value would be negative e.g. $\mathbf{u_2} = -2\text{ms}^{-1}$).

Restitution

When two objects collide, the speeds at which they rebound depends on the material they are made of (how "bouncy" it is). *Newton's Law of Restitution* or *Newton's Experimental Law* states that:

$$e = \frac{\text{Speed of Separation}}{\text{Speed of Approach}}$$

where e is the coefficient of restitution.

Some text books put in symbols for the relative speeds (such as $u_1 - u_2$ etc.) but I find that I am more likely to make a sign error if I use a formula like this rather than the more wordy one above. Sign errors are very easy to make in this type of question!

The coefficient of restitution must lie in the range $0 \leqslant e \leqslant 1$. When $e = 1$ the collision is *perfectly elastic* and in this case no kinetic energy is lost. If $e = 0$ then the collision is *perfectly inelastic* and the two particles *coalesce* i.e. they stick together and move as one after the collision.

Most collision questions will involve you using these two equations. It is helpful if you write "Momentum $\implies$" and "Restitution $\implies$" (or "NEL $\implies$") next to the equations as you write them down.

Oblique impacts (where you need to consider two perpendicular directions) are covered in the STEP 3 specification.

Multiple Collisions

STEP questions often involve multiple collisions (sometimes where there are several objects which collide in turn, sometimes where two objects bounce back off a fixed surface and collide again). The key thing here is to find a notation that you can use and won't get confused by! It is also helpful if you write down what the notation you are using means.

For example, if I had a situation with three objects then I might use:

u_0, v_0, w_0	initial velocities
u_1, v_1, w_1	velocities after the first collision
u_2, v_2, w_2	velocities after the second collision etc.

Note that here some of these will be the same (if the first collision was between the first two particles then we will have $w_0 = w_1$), and some may be equal to 0.

A different approach would be to call the initial velocities u_1, u_2, u_3 then the velocities after the first collision v_1, v_2, v_3 etc. It is very important to explain what notation you are using so that your method can be followed more easily.

Hooke's Law

Hooke's Law for extensible strings and springs states that $T = kx = \frac{\lambda x}{l}$, where k is the stiffness, λ is the modulus of elasticity, l is the natural length of the string or spring and x is the extension of the spring or string. In the case of a spring x could instead be the amount the spring is *compressed* by.

The energy in an extended string, or an extended or compressed spring, is given by $E = \frac{\lambda x^2}{2l}$.