STEP Support Programme

STEP 3 Calculus Topic Notes

Maclaurin's Series

The Maclaurin's series for $\mathrm{f}(x)$ is given by:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \frac{x^3}{3!}\mathrm{f}'''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

The Maclaurin's series that you are expected to know are:

$$\begin{aligned}
\mathrm{e}^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots
\end{aligned}$$

Lengths of curves

In the last 20 years, if a question has needed an arc length formulae then it has been given in the question (you might be asked to convert from the first form below to the second form).

The arc length along a curve $y = \mathrm{f}(x)$ from $x = a$ and $x = b$ is given by:

$$\int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \qquad \text{or}$$
$$\int_a^b \sqrt{1 + (\mathrm{f}'(x))^2}\,\mathrm{d}x\,.$$

Or if you have parametric equations $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$:

$$\int_{t=\alpha}^{t=\beta} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$$

You can find an explanation of the arc length formula from Khan Academy.

Volumes of revolution

The volume of revolution that a curve $y = \mathrm{f}(x)$ makes as it is rotated about the x-axis between $x = a$ and $x = b$ is:

$$\pi \int_a^b \big(\mathrm{f}(x)\big)^2 \,\mathrm{d}x \quad \text{or} \quad \pi \int_a^b y^2 \,\mathrm{d}x$$

This formula is derived from considering lots of small discs of radius $\mathrm{f}(x)$ and height δx and considering what happens to the sum of these as $\delta x \to 0$.

In a similar way, the volume of revolution about the y-axis is $\pi \int x^2 \,\mathrm{d}y$.

Mean value of a function

The mean value of a function $\mathrm{f}(x)$ in the interval $a \leqslant x \leqslant b$ is given by:

$$\bar{\mathrm{f}} = \frac{1}{b-a} \int_a^b \mathrm{f}(x) \,\mathrm{d}x$$

You can find an explanation of where this formula comes from here.