STEP Support Programme

STEP 2 Vectors Questions

1 **2002 S2 Q7**

In 3-dimensional space, the lines m_1 and m_2 pass through the origin and have directions $\mathbf{i}+\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$, respectively. Find the directions of the two lines m_3 and m_4 that pass through the origin and make angles of $\pi/4$ with both m_1 and m_2. Find also the cosine of the acute angle between m_3 and m_4.

The points A and B lie on m_1 and m_2 respectively, and are each at distance $\lambda\sqrt{2}$ units from O. The points P and Q lie on m_3 and m_4 respectively, and are each at distance 1 unit from O. If all the coordinates (with respect to axes $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$) of A, B, P and Q are non-negative, prove that:

(i) there are only two values of λ for which AQ is perpendicular to BP;

(ii) there are no non-zero values of λ for which AQ and BP intersect.

2 **2011 S2 Q5**

The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to an origin O, and O, A and B are non-collinear. The point C, with position vector $\mathbf{c}$, is the reflection of B in the line through O and A. Show that $\mathbf{c}$ can be written in the form

$$\mathbf{c} = \lambda\mathbf{a} - \mathbf{b}$$

where $\lambda = \dfrac{2\,\mathbf{a}.\mathbf{b}}{\mathbf{a}.\mathbf{a}}$.

The point D, with position vector $\mathbf{d}$, is the reflection of C in the line through O and B. Show that $\mathbf{d}$ can be written in the form

$$\mathbf{d} = \mu\mathbf{b} - \lambda\mathbf{a}$$

for some scalar μ to be determined.

Given that A, B and D are collinear, find the relationship between λ and μ. In the case $\lambda = -\frac{1}{2}$, determine the cosine of $\angle AOB$ and describe the relative positions of A, B and D.

3 2009 S2 Q8

The non-collinear points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, respectively. The points P and Q have position vectors $\mathbf{p}$ and $\mathbf{q}$, respectively, given by

$$\mathbf{p} = \lambda\mathbf{a} + (1-\lambda)\mathbf{b} \quad \text{and} \quad \mathbf{q} = \mu\mathbf{a} + (1-\mu)\mathbf{c}$$

where $0 < \lambda < 1$ and $\mu > 1$. Draw a diagram showing A, B, C, P and Q.

Given that $CQ \times BP = AB \times AC$, find μ in terms of λ, and show that, for all values of λ, the the line PQ passes through the fixed point D, with position vector $\mathbf{d}$ given by $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$. What can be said about the quadrilateral $ABDC$?

4 2010 S2 Q5

The points A and B have position vectors $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $5\mathbf{i} - \mathbf{j} - \mathbf{k}$, respectively, relative to the origin O. Find $\cos 2\alpha$, where 2α is the angle $\angle AOB$.

(i) The line L_1 has equation $\mathbf{r} = \lambda(m\mathbf{i} + n\mathbf{j} + p\mathbf{k})$. Given that L_1 is inclined equally to OA and to OB, determine a relationship between m, n and p. Find also values of m, n and p for which L_1 is the angle bisector of $\angle AOB$.

(ii) The line L_2 has equation $\mathbf{r} = \mu(u\mathbf{i} + v\mathbf{j} + w\mathbf{k})$. Given that L_2 is inclined at an angle α to OA, where $2\alpha = \angle AOB$, determine a relationship between u, v and w.

Hence describe the surface with Cartesian equation $x^2 + y^2 + z^2 = 2(yz + zx + xy)$.