# STEP Support Programme

## Assignment 2

## Warm-up

**1** **(i)** Simplify $(2x-3)^2-(x-1)^2$, giving your answer in factorised form. Check your answer by evaluating it for $x=1$ and $x=2$.

**(ii)** Simplify

$$\frac{x}{x^2-y^2}-\frac{y}{(x-y)^2}-\frac{1}{x+y}.$$

Hence find the possible values of $x$ and $y$ for which $\dfrac{x}{x^2-y^2}-\dfrac{y}{(x-y)^2}-\dfrac{1}{x+y}=0$.

**(iii)** Show that

$$\sqrt{1+x^2}\ -x=\frac{1}{\sqrt{1+x^2}\ +x}.$$

Deduce that if $x$ is very large, then $\sqrt{1+x^2}-x$ is approximately equal to $\dfrac{1}{2x}$.

**(iv)** Simplify $(x^2-\sqrt{2}\,x+1)(x^2+\sqrt{2}\,x+1)$. Hence find the solutions to $x^4+1=0$.

Your answer will involve $\sqrt{-1}$ (written as i).

# Preparation

**2** **(i)** Sketch the line $y = x + 1$ for $-2 \leqslant x \leqslant 2$.

What is the greatest value of $x + 1$ in this range?

**(ii)** Sketch the line $y = -2x + c$ for $-2 \leqslant x \leqslant 2$.

Show that the greatest value of $-2x + c$ in this range is $4 + c$. What is the least value?

**(iii)** Sketch $y = mx + 1$ for $-2 \leqslant x \leqslant 2$ in the cases $m > 0$, $m = 0$ and $m < 0$.

What are the greatest and least values of $mx + 1$ in each case?

**(iv)** Sketch the curve (parabola) $y = (x - 1)^2$ for $-2 \leqslant x \leqslant 2$.

What are the greatest and least values of $(x - 1)^2$ in this range?

Be careful here: the minimum value is **not** at one of the end points.

**(v)** Sketch the curve $y = (x - 3)^2$. What are the greatest and least values of $(x - 3)^2$ for $-2 \leqslant x \leqslant 2$?

**(vi)** Write the expression $x^2 - 8x + 21$ in the form $(x + a)^2 + b$. Hence sketch the curve $y = x^2 - 8x + 21$ and find the greatest and least values of $x^2 - 8x + 21$ in the range $0 \leqslant x \leqslant 5$.

**(vii)** Sketch the curve $y = x^2 + 2kx$ for $-2 \leqslant x \leqslant 2$, where $-2 < k < 2$.

What are the greatest and least values of $x^2 + 2kx$ for $-2 \leqslant x \leqslant 2$?

What would your answers be if $k > 2$?

Use the same techniques as in part **(vi)** to help you sketch the curve.

## The STEP question

**3** **(i)** Find the greatest and least values of $bx + a$ for $-10 \leqslant x \leqslant 10$, distinguishing carefully between the cases $b > 0$, $b = 0$ and $b < 0$.

**(ii)** Find the greatest and least values of $cx^2 + bx + a$, where $c \geqslant 0$, for $-10 \leqslant x \leqslant 10$, distinguishing carefully between the cases that can arise for different values of $b$ and $c$.

## Discussion

This question has some features that are very typical of STEP questions.

First is the use of letters, $a$, $b$ and $c$ in this case, rather than numbers to make the equations less specific. The correct term for these letters is *parameters*, whereas $x$ is a *variable*. In each case you will have to give your answers in terms of the given parameters.

Second, although you are not told to draw sketches, you should do so: it is much easier to work on the different cases if you have sketches in front of you. Think back to question **2** to help you.

## Warm down

**4** Five children (Ahmed, Bachendri, Charlie, Daniel and Emily) raced each other. First they raced to the spreading chestnut tree, and then they raced back to their starting point. The following facts are known:

**(i)** Ahmed was fourth in the race to the tree.

**(ii)** The person who was last to the tree managed to win the race back.

**(iii)** The person who won the race to the tree was third on the way back.

**(iv)** The person who was third in the race to the tree was second on the way back.

**(v)** Bachendri was fourth on the way back.

**(vi)** Charlie reached the tree before Daniel.

**(vii)** Charlie got back to the start before Emily.

For each race (to the tree and back again), write down the order in which the children finished.

You might want to use a table to display the results, giving names such as Zak (or letters such as $Z$) to people you do not yet know but about whom you have information. (So from part (iv), you could say that $Z$ came 3rd in the first race and 2nd in the second race.)