STEP Support Programme

STEP 1 Specification Pure Notes

These notes are designed to help students in preparing for STEP 1. They cover the "bold and italic" sections of the STEP 1 specification which are not covered in the A-level single Mathematics specifications. Many of these topics will be covered in A and AS level Further Mathematics.

Proof

Proof by Induction

Proof by induction is a method of proving a proposition P(n) for natural numbers $n = 1, 2, 3, \cdots$ (or possibly $n = 0, 1, 2, \cdots$ or even $n = 5, 6, 7, \cdots$).

The way of doing this is to show that if it is true for a value of n, then it is also true for the next value of n. Then if it is true when $n = 1$ then it will be true for $n = 2$, and since it is true for $n = 2$ it will be true for $n = 3$ and continuing this argument tells is that it is true for $n = 1, 2, 3, \cdots$. The usual analogy given is falling dominoes — if you push the first one over all the rest will fall one after another.

A proof by induction follows this structure:

- Prove that the proposition is true when $n = 1$ (i.e. show that P(1) is true).
- Assume that the proposition is true when $n = k$ (i.e. assume that P(k) is true).
- Show that if you assume that P(k) is true then P($k + 1$) is also true (i.e. if the proposition is true when $n = k$ then it is true for $n = k + 1$).
- Finish the proof with a sentence like "Hence if the proposition is true when $n = k$ then it is true when $n = k + 1$ and since it is true for $n = 1$ it is true for all integers $n \geqslant 1$".

Note that you could show that P(1) is true after showing the induction step $\text{P}(k) \implies \text{P}(k+1)$.

An example:

$$\text{Prove that } \sum_{i=1}^{n} i^2 = \tfrac{1}{6}n(n+1)(2n+1) \text{ for all integers } n \geqslant 1.$$

I often find it helpful to expand the sum to see more clearly what I am being asked to do. In this case we are being asked to show that $1^2 + 2^2 + 3^3 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

- **Base case:** When $n = 1$ we have:

$$1^2 = \frac{1}{6} \times 1 \times (1+1) \times (2+1)$$
$$1 = \frac{1}{6} \times 6$$

  which is true, and so the proposition is true when $n = 1$.

- **Inductive step:** We now assume that the proposition is true when $n = k$, which means we are assuming that:

$$1^2 + 2^2 + 3^3 + \cdots + k^2 = \frac{1}{6}k(k+1)(2k+1)$$

  We want to show that if P(k) is true then P($k + 1$) is also true, i.e. we are RTP[1] that $\sum_{i=1}^{k+1} i^2 = \frac{1}{6}(k+1)\big[(k+1)+1\big]\big[2(k+1)+1\big]$

[1]Required to prove

We have:

$$
\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= 1^2 + 2^2 + \cdots + k^2 + (k+1)^2 \\
&= \tfrac{1}{6}k(k+1)(2k+1) + (k+1)^2 && \text{using } \mathrm{P}(k) \\
&= \tfrac{1}{6}(k+1)\Big[k(2k+1) + 6(k+1)\Big] \\
&= \tfrac{1}{6}(k+1)\Big[2k^2 + 7k + 6\Big] \\
&= \tfrac{1}{6}(k+1)\Big[(k+2)(2k+3)\Big] \\
&= \tfrac{1}{6}(k+1)\big[(k+1)+1\big]\big[2(k+1)+1\big] && \text{as required.}
\end{aligned}
$$

Notice that I factorised out the $\frac{1}{6}$ and $(k+1)$ terms rather than expanding everything. This means that I didn't have to factorise a cubic later.

- **Completion** If the proposition is true when $n = k$, then it is also true when $n = k + 1$, and since it is true when $n = 1$ it is true for all integers $n \geqslant 1$.

Note that the base case might not be $n = 1$. For example, you might be asked to prove that $n! > 2^n$ for $n \geqslant 4$, where the base case is $n = 4$.

There are more notes on proof by induction in Foundation Assignment 20. You can also find STEP questions involving proof by induction by going to the STEP Database and searching for "proof by induction".

If and only if

"If and only if" means that an *implication* can work in either direction, for example a quadratic equation has a repeated root if and only if the discriminant is equal to zero.

Not every implication can be reversed, for example $a = b \implies a^2 = b^2$ but we cannot write this the other way. We could write $a^2 = b^2$ **if** $a = b$ (or less usefully we could say $a = b$ only if $a^2 = b^2$).

There is more on "if and only if" in Foundation Assignment 10.

Necessary and Sufficient

"Necessary and sufficient" is essentially the same as "If and only if". A necessary and sufficient condition for a quadratic to have a repeated root is that the discriminant is equal to zero.

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ is a necessary condition for a point to be a point of inflection, but it is not a sufficient condition (for example the curve $y = x^4$ has $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ when $x = 0$, but this is a minimum not a point of inflection). There is more about points of inflection in Foundation Assignment 13.

$x = 3$ is a sufficient condition for $x^2 = 9$, but it is not necessary (as we could have $x = -3$).

Being written in red ink is neither a necessary nor sufficient condition for a number to be prime.

You can find more on necessary and sufficient conditions on pages 45-48 of this guide to logic.

Algebra and functions

Inequalities

When trying to solve inequalities you do need to be careful. For example if trying to solve $x + \frac{1}{x} > 2$ you might be tempted to multiply throughout by x — but since x could be negative this is A Bad Idea[2].

A perhaps better approach is to sketch a graph of $y = x + \frac{1}{x}$, add on $y = 2$ and solve $x + \frac{1}{x} = 2$ to find where the lines meet. You can then use your sketch to solve the inequality.

This approach can be extended to any sort of function. There is more on inequalities in the STEP 2 Equations and Inequalities module.

The Remainder Theorem

The remainder theorem states that when we divide a polynomial $\mathrm{p}(x)$ by $(x-a)$ then the remainder is $\mathrm{p}(a)$.

When we divide a polynomial $\mathrm{p}(x)$ by a linear factor $(x-a)$ then we can write $\mathrm{p}(x) = (x-a)\mathrm{q}(x)+r$ where $\mathrm{q}(x)$ is the *quotient* (and has degree one less than $\mathrm{p}(x)$, and r is the remainder — which will be a constant).

Then we have:

$$\begin{aligned} \mathrm{p}(x) &= (x-a)\mathrm{q}(x) + r \\ \implies \mathrm{p}(a) &= (a-a)\mathrm{q}(a) + r \\ \implies \mathrm{p}(a) &= r \end{aligned}$$

Hence when a polynomial $\mathrm{p}(x)$ is divided by $(x-a)$ the remainder is $\mathrm{p}(a)$. Using a similar method you can show that show that when $\mathrm{p}(x)$ is divided by $(bx-a)$ the remainder is $\mathrm{p}(\frac{a}{b})$.

The remainder theorem is closely related to the factor theorem; $(x-a)$ is a factor of $\mathrm{p}(x)$ if and only if $\mathrm{p}(a) = 0$. This is an "if and only if", so a little care is needed with the proof.

If $(x-a)$ is a factor then we have $\mathrm{p}(x) = (x-a)\mathrm{q}(x)$, and substituting $x = a$ gives $\mathrm{p}(a) = 0$. Hence $(x-a)$ is a factor of $\mathrm{p}(x) \implies \mathrm{p}(a) = 0$.

If $\mathrm{p}(a) = 0$ then we have:

$$\begin{aligned} \mathrm{p}(x) &= (x-a)\mathrm{q}(x) + r \\ \mathrm{p}(a) &= (a-a)\mathrm{q}(a) + r \\ \mathrm{p}(a) &= r \\ \implies 0 &= r \\ \implies \mathrm{p}(x) &= (x-a)\mathrm{q}(x) \end{aligned}$$

and hence we have $\mathrm{p}(a) = 0 \implies (x-a)$ is a factor of $\mathrm{p}(x)$.

[2] It is possible to solve the equation like this, but positive and negative x values need to be considered separately. Care is needed to make sure that any solutions are valid for the range of values of x you are currently considering.

A possibly useful fact is that if polynomial $\mathrm{p}(x)$ has a remainder of b when divided by $(x-a)$, then the polynomial $\mathrm{p}(x)-b$ has a factor of $(x-a)$.

Proof:

$$\begin{aligned}&\text{We have } \mathrm{p}(a)=b\\&\text{Consider } \mathrm{p}(x)-b\\&\text{Then } \mathrm{p}(a)-b=b-b=0\\&\text{Hence } (x-a) \text{ is a factor of } \mathrm{p}(a)-b\end{aligned}$$

Equating Coefficients (Including roots of quadratics and algebraic division)

If we have an identity connecting two polynomials then the coefficients of the corresponding powers of x are equal.

For example, if the roots of the quadratic x^2+bx+c are $x=\alpha$ and $x=\beta$ then we have:

$$\begin{aligned}x^2+bx+c &\equiv (x-\alpha)(x-\beta)\\x^2+bx+c &\equiv x^2-(\alpha+\beta)x+\alpha\beta\end{aligned}$$

And so we have $b=-(\alpha+\beta)$ and $c=\alpha\beta$.

You can use this idea of equating coefficients to divide polynomials by quadratic and higher degree expressions[3].

For example, if you wanted to divide the polynomial $3x^4+7x^3+2x^2+15x+6$ by x^2+3x+1. The remainder would be of the form $px+q$ so you would have something like:

$$3x^4+7x^3+2x^2+15x+6 \equiv (x^2+3x+1)(ax^2+bx+c)+px+r$$

Equating coefficients gives:

$$\begin{aligned}x^4: &\quad 3=a\\x^3: &\quad 7=3a+b\\x^2: &\quad 2=a+3b+c\\x^1: &\quad 15=3c+b+p\\\text{constant}: &\quad 6=c+r\end{aligned}$$

Then working down the equations we have $a=3$, $b=-2$, $c=5$, $p=2$ and $r=1$ and so we have

$$3x^4+7x^3+2x^2+15x+6 \equiv (x^2+3x+1)(3x^2-2x+5)+2x+1$$

Functions

A *function* is a relationship or mapping which associates each element x of a set X (the *domain* of the function) to an element y of a set Y (the *codomain* of the function). Note that X and Y might be the same set. The *range* of a function is the set of possible values of y (given the range X), so the range is a subset of the codomain.

For example, if $\mathrm{f}(x)=x^2$ then the domain and the codomain could be the integers ($\mathbb{Z}$), but if the domain is $\mathbb{Z}$, then the range will be non-negative integers.

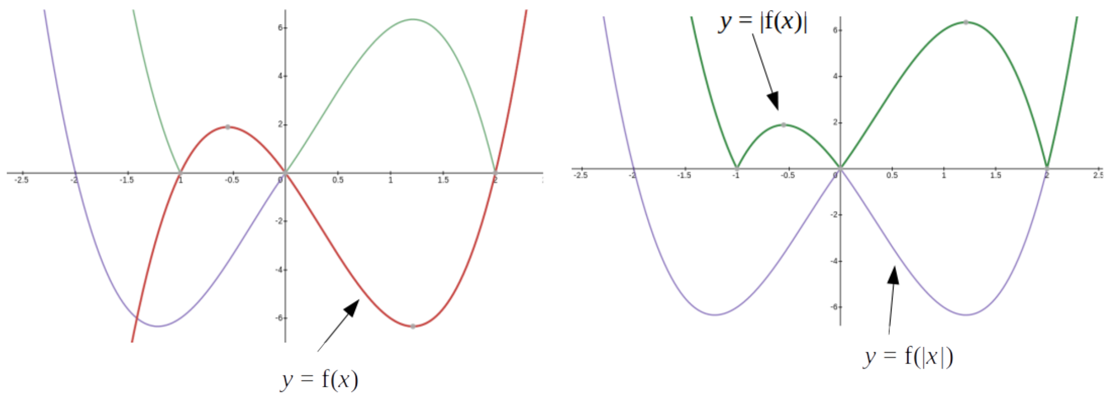
[3] You can use algebraic long division, but it does get a little cumbersome.

Graphs of modulus functions

Consider a curve $y = \mathrm{f}(x)$. The graph of $y = |\mathrm{f}(x)|$ will be the same in the regions that lie above the x axis (where $\mathrm{f}(x) \geqslant 0$) and any parts that lie below the x axis (where $\mathrm{f}(x) < 0$) will be reflected about the x axis so that they lie above the x axis (I think of them as being "snapped over").

The graph of $y = \mathrm{f}(|x|)$ will be the same as $y = \mathrm{f}(x)$ for all positive values of x and then it will have reflection symmetry about the y axis.

The pictures below illustrate these for a cubic.



You might like to try and sketch $y = |\sin x|$ and $y = \sin |x|$. Desmos can be used to check your answer. Note that if $\mathrm{f}(x)$ is an even function then $\mathrm{f}(-x) = \mathrm{f}(x)$ and hence $y = \mathrm{f}(|x|)$ will be the same as $y = \mathrm{f}(x)$.

Limits of functions

When sketching functions you might need to consider what happens as $x \to \infty$, or as x tends to a value for which the function is undefined.

Consider the graph $y = \dfrac{x}{(x-1)^2}$. This is undefined for $x = 1$, and as x gets close to 1, y gets very large. For this function y will be positive whenever x is positive, so as $x \to 1_-$[4] and as $x \to 1_+$[5] we have $y \to +\infty$.

Here I have specifically shown that y is positive in the region near $x = 1$. Sometimes you might have $y \to -\infty$.

As $x \to \pm\infty$ we can write $y = \dfrac{1}{x - 2 + \frac{1}{x}}$, which shows that y gets very small as $x \to \pm\infty$. Noting that y is positive when x is positive and y is negative when x is negative we have $y \to 0_+$ as $x \to +\infty$ and $y \to 0_-$ as $x \to -\infty$.

The work done here, together with what happens when $x = 0$, is enough to enable you to draw a sketch of the graph. Have a go and then check it with Desmos.

See also the section on limits under *Sequences and Series* on pages 7 and 8.

[4] This notation means as x tends to 1 "from below", so for values of x less than 1.

[5] x tends to 1 "from above".

Coordinate Geometry

The circle theorems in this section which are in bold italics are all included in Higher Level GCSE specifications, even if they are not all included in the A-level specifications. There are lots of revision guides on circle theorems available online.

Sequences and Series

The *binomial coefficients* have various representations including $n\text{C}r$, ${}^n\text{C}_r$ and $\binom{n}{r}$. The algebraic form is given by:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Where $n! = 1 \times 2 \times 3 \times \cdots \times n$.

Arrangements

Permutations are arrangements in which the order matters. For example: 1st, 2nd and 3rd places are awarded for a colouring competition in which 10 people entered. The number of ways of picking a 1st place entry is 10, and then there are 9 ways of picking 2nd place and 8 ways of picking 3rd place. The total number of ways of allocating the 3 prizes is:

$$10 \times 9 \times 8 = \frac{10!}{7!}$$

In general, the number of ways of picking r objects from a total of n where the order of choosing matters is:

$${}^n\text{P}_r = \frac{n!}{(n-r)!}$$

If instead we wanted to pick a team of 3 people from a group of 10 the order wouldn't matter, i.e. a team of (Alice, Bob, Charlie) would be the same as a team of (Charlie, Bob, Alice). There are 3! ways of arranging Alice, Bob and Charlie which still make up the same team, so we take the number of permutations of 3 from 10 and divide this by 3! to get $\dfrac{10!}{7! \times 3!}$.

In general, the number of combinations of r objects from a group of n is:

$${}^n\text{C}_r = \frac{n!}{(n-r)!r!}$$

${}^n\text{C}_r$ is sometimes called "n Choose r".

Limits

A sequence, such as $x_n = \text{f}(n)$ or $x_{n+1} = \text{f}(x_n)$ might converge to a limit.

For example, the sequence $x_n = 1 - \dfrac{1}{n}$ (for $n \geqslant 1$) gives the terms $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots$. As n increases the terms of the sequence get closer and closer to 1. We say that $x_n \to 1$ as $n \to \infty$.

Sometimes we might need to do a bit of manipulation when trying to find a limit. Consider the sequence $x_n = \dfrac{2n+1}{3n-2}$. Substituting some values of n suggests that this sequence is tending to something as n gets large.

If we divide throughout by n we get $x_n = \dfrac{2+\frac{1}{n}}{3-\frac{2}{n}}$, and as n gets very large $\frac{1}{n}$ and $\frac{2}{n}$ tend to zero. Hence as $n \to \infty$, $x_n \to \frac{2}{3}$.

There is some more on limits in Foundation Module 15, and also in question 5 of the Mixed Pure STEP 3 module.

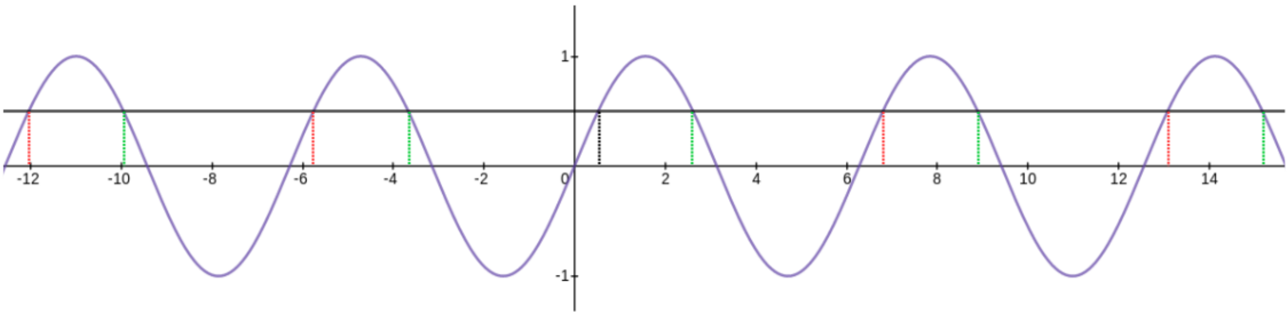
Trigonometry

General solutions

Usually in A-level you will be asked to give all the solutions to a trig equation in a given range. In STEP you might be asked to find all the solutions, i.e. the general form of the solution.

For this you need to consider the graph of the relevant trig function (you can memorise the general solution forms if you like, but personally I don't like to rely on the accuracy of my memory in these cases!).

Consider $\sin x = \frac{1}{2}$.



The principal solution (the one your calculator will give) is $x = \frac{1}{6}\pi$. This is the one represented by the black dotted line. The graph of $y = \sin x$ is periodic, and repeats every 2π. This means that all of the red lines can be represented by $x = \frac{1}{6}\pi + 2n\pi$ for $n = \cdots, -2, -1, 0, 1, 2, \cdots$.

The green solutions come from the symmetry of $y = \sin x$. The first one will be equal to $\pi - \frac{1}{6}\pi$ and so they can be found from $x = \pi - \frac{1}{6}\pi + 2n\pi = \frac{5}{6}\pi + 2n\pi$.

If you wanted to solve $\sin\left(3x + \frac{\pi}{5}\right) = \frac{1}{2}$ then you would set $3x + \frac{\pi}{5} = \frac{1}{6}\pi + 2n\pi$ and $3x + \frac{\pi}{5} = \pi - \frac{1}{6}\pi + 2n\pi$ then solve these for x.

You can use a similar method to find the general solutions for other trig equations. These are:

- If $\theta = \alpha$ is a solution of $\cos x = k$ then so are $\theta = \alpha + 2n\pi$ and $\theta = -\alpha + 2n\pi$
- If $\theta = \alpha$ is a solution of $\tan x = k$ then so are $\theta = \alpha + n\pi$
- If $\theta = \alpha$ is a solution of $\sin x = k$ then so are $\theta = \alpha + 2n\pi$ and $\theta = (\pi - \alpha) + 2n\pi$

Exponentials and Logarithms

Change of base

The change of base formula is:

$$\log_a x = \frac{\log_b x}{\log_b a}\,.$$

To show this, start by using $\log_a x = y$. Then we have:

$$\begin{aligned} \log_a x &= y \\ x &= a^y \\ \log_b x &= y \log_b a \\ \frac{\log_b x}{\log_b a} &= y \end{aligned}$$

Equating expressions for y then gives $\log_a x = y$.

Differentiation

Continuity and Differentiability

A *continuous function* is one that can be drawn without the pencil leaving the paper.

Examples of ways in which a graph might not be continuous include jumps (such as in $y = \lfloor x \rfloor$) and vertical asymptotes (e.g. $y = \frac{1}{x}$).

The formal definition for a function to be continuous is:

For every value c in the domain of $\mathrm{f}(x)$, $\mathrm{f}(c)$ defined
and
we have both $\lim\limits_{x \to c_-} \mathrm{f}(x) = \mathrm{f}(c)$ and $\lim\limits_{x \to c_+} \mathrm{f}(x) = \mathrm{f}(c)$

This last part means that if we head towards $x = c$ from either below or above c, the function tends to the value $\mathrm{f}(c)$. The curve "joins up" everywhere.

Restricting the domain of a function may mean that it is then continuous. For example, $y = \frac{1}{x}$ **is** continuous on $x > 0$.

A *differentiable function* is "smooth" everywhere — which means that if we zoom into any part of the graph sufficiently then it will look like a straight line. A differentiable function must be a continuous one.

$y = |x|$ is an example of a function which is continuous, but not differentiable (the bit of the graph around $y = x$ will remain "pointy" no matter how much you zoom in).

The formal definition of a differentiable function is that, for every value of x in the functions domain, we have:

$$\lim_{h \to 0_-} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h} = \lim_{h \to 0_+} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Please note that for STEP you only need have an informal understanding of these!

Differentiation of other Trigonometric functions

We have:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)\\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \quad \text{(using the quotient rule)}\\ &= -\operatorname{cosec}^2 x\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) &= \frac{\mathrm{d}}{\mathrm{d}x}(\cos x)^{-1}\\ &= -(\cos x)^{-2} \times -\sin x\\ &= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}\\ &= \tan x \sec x\end{aligned}$$

Note that $(\cos x)^{-1} = \dfrac{1}{\cos x}$ is very different to $\cos^{-1} x$ which is the inverse function.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosec} x) &= \frac{\mathrm{d}}{\mathrm{d}x}(\sin x)^{-1}\\ &= -(\sin x)^{-2} \times \cos x\\ &= -\frac{\cos x}{\sin x} \times \frac{1}{\sin x}\\ &= -\cot x \operatorname{cosec} x\end{aligned}$$

All of these I tend to derive rather than remember.

Integration

Integrability

A function is *integrable* if the integral is *well defined* — that is it makes some sort of sense!

A function does not have to be differentiable, or even continuous, to be integrable. The integral $\displaystyle\int_0^3 \lfloor x \rfloor$ will give the area of three rectangles of height 0, 1, and 2 so we have $\displaystyle\int_0^3 \lfloor x \rfloor = 3$.

The function $y = \dfrac{1}{x^2}$ is not integrable everywhere due to the asymptote at $x = 0$.

There is more on integrals of functions where the integrand is undefined somewhere in the range of integration in the STEP 2 Specification Pure notes.

Integration by inspection

Quite often a tricky-looking integrand can be integrated by "having a guess" and then differentiating to check and see what constants you might need to insert.

For example, consider $\displaystyle\int \frac{5x^4 + x^2}{\sqrt[3]{3x^5 + x^3}}\,\mathrm{d}x$.

We could write the integrand here as $(5x^4+x^2)(3x^5+x^3)^{-\frac{1}{3}}$. This suggests that it might be helpful to see what happens if we differentiate $(3x^5 + x^3)^{\frac{2}{3}}$.

We have:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(3x^5 + x^3)^{\frac{2}{3}} &= (15x^4 + 3x^2) \times \tfrac{2}{3}(3x^5 + x^3)^{-\frac{1}{3}}\\ &= (10x^4 + 2x^2)(3x^5 + x^3)^{-\frac{1}{3}}\end{aligned}$$

Which is twice the integrand we started with. This means that we have:

$$\int \frac{5x^4 + x^2}{\sqrt[3]{3x^5 + x^3}}\,\mathrm{d}x = \tfrac{1}{2}(3x^5 + x^3)^{\frac{2}{3}} + c$$

Partial fractions with repeated linear factors

In general, a fraction of the form $\dfrac{ax+b}{(x-c)^2}$ can be split up into the fractions $\dfrac{A}{x-c}+\dfrac{B}{(x-c)^2}$.

Note that a and A can represent different numbers.

Example: Find $\displaystyle\int \frac{2x+1}{(x+1)(x-1)^2}\,\mathrm{d}x$.

$$\begin{aligned}\frac{2x+1}{(x+1)(x-1)^2} &= \frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}\\ &= \frac{A(x-1)^2}{(x+1)(x-1)^2}+\frac{B(x+1)(x-1)}{(x+1)(x-1)^2}+\frac{C(x+1)}{(x+1)(x-1)^2}\\ &= \frac{(A+B)x^2+(-2A+C)x+(A-B+C)}{(x+1)(x-1)^2}\end{aligned}$$

Equating coefficients gives:

$$\begin{aligned} x^2: &\quad A+B=0\\ x: &\quad C-2A=2\\ \text{Constant}: &\quad A-B+C=1\end{aligned}$$

Solving these simultaneously gives:

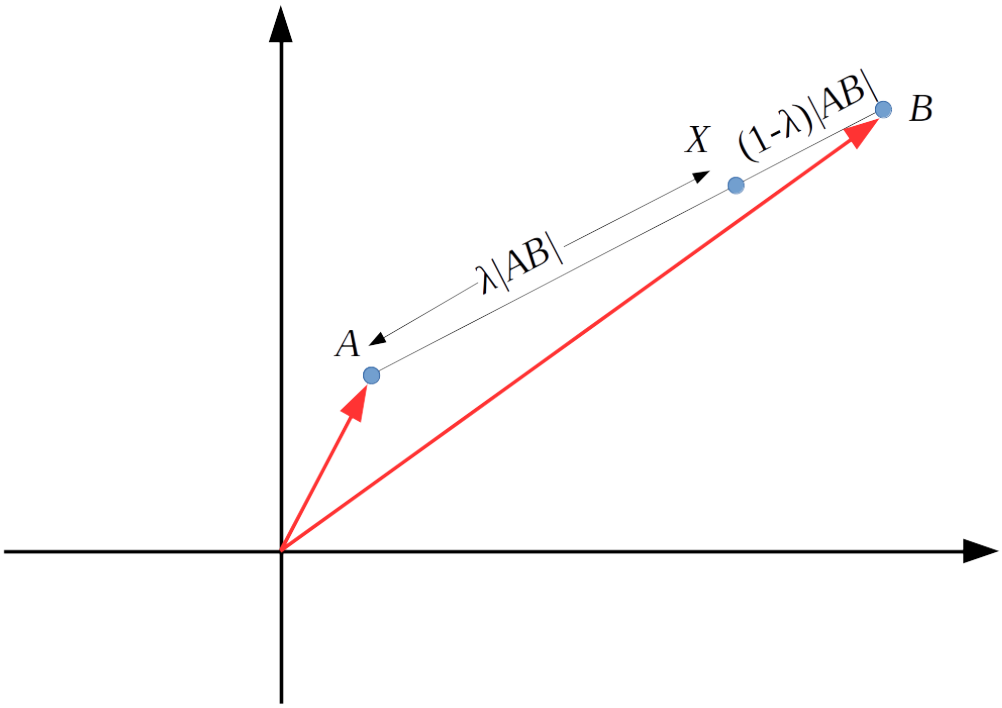
$$A=-\tfrac{1}{4}, B=\tfrac{1}{4}, C=\tfrac{3}{2}$$

We then have:

$$\int -\frac{1}{4(x+1)}+\frac{1}{4(x-1)}+\frac{3}{2(x-1)^2}\,\mathrm{d}x = -\frac{1}{4}\ln(x+1)+\frac{1}{4}\ln(x-1)-\frac{3}{2}(x-1)^{-1}+c$$

Vectors

Suppose that point X lies on the line AB such that $|AX| : |XB| = \lambda : 1 - \lambda$. This situation is shown in the picture below:



The position vector of X can be found by:

$$\begin{aligned}\overrightarrow{OX} &= \overrightarrow{OA} + \overrightarrow{AX} \\ &= \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) \\ &= (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}\end{aligned}$$

A quick sanity check shows that if we take $\lambda = 0$ then point X is at A, and if we take $\lambda = 1$ then point X is at B, which is reassuring.

In the diagram X is shown as lying between A and B. X can actually lie anywhere on the extended line AB — if $\lambda < 0$ or if $\lambda > 1$ then X will not be between A and B.