STEP Support Programme

Mechanics STEP Questions

This is a selection of old STEP 1 and STEP 2 questions which are based on content from the new (2019 onwards) STEP 1 specification.

2012 S1 Q11

1 Preparation

(i) If $\tan\theta = \frac{5}{12}$ and $0 \leqslant \theta < \frac{\pi}{2}$, find $\cos\theta$ and $\sin\theta$.

(ii) A block of mass 20kg is lying on a slope which makes an angle of 30° to the horizontal. The coefficient of friction between the block and the slope is 0.7. Take $g = 10$. Find the minimum value needed to:

(a) pull the block up the slope;

(b) push the block down the slope.

Leave your answers in surd form.

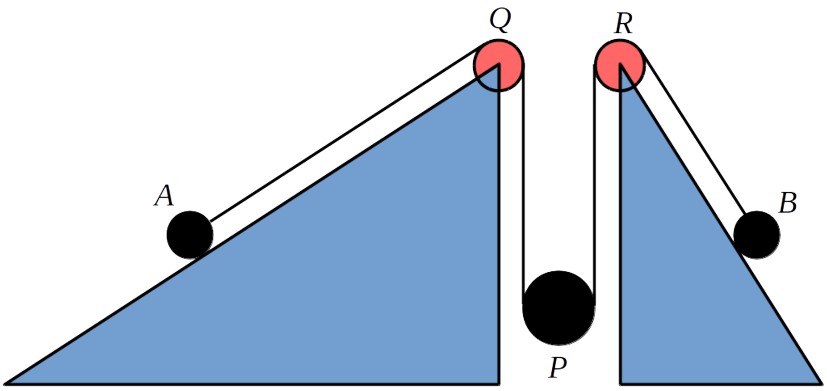
Draw a good big diagram, putting in all the forces! Remember that friction counteracts the tendency to move.

(iii) Two weights, with masses m_1 and m_2 where $m_1 > m_2$, are connected by a light inextensible string over a smooth pulley. Find the acceleration of the weights in terms of m_1, m_2 and g.

2 The Mechanics question

The diagram shows two particles, A of mass $5m$ and B of mass $3m$, connected by a light inextensible string which passes over two smooth, light, fixed pulleys, Q and R, and under a smooth pulley P which has mass M and is free to move vertically.

Particles A and B lie on fixed rough planes inclined to the horizontal at angles of $\arctan\frac{7}{24}$ and $\arctan\frac{4}{3}$ respectively. The segments AQ and RB of the string are parallel to their respective planes, and segments QP and PR are vertical. The coefficient of friction between each particle and its plane is μ.



(i) Given that the system is in equilibrium, with both A and B on the point of moving up their planes, determine the value of μ and show that $M = 6m$.

(ii) In the case when $M = 9m$, determine the initial accelerations of A, B and P in terms of g.

Discussion

Note that the second part of the question the three particles can have different accelerations (unlike question **1(iii)**) but they are related because the length of the string is fixed.

2010 S1 Q10

3 Preparation

This mechanics question only requires fairly simple manipulations (differentiation, scalar product) of vectors, so there is not really any preparation to be done; just get on with it!

Of course, to find the derivative of a vector, you just differentiate its components.

4 The Mechanics question

A particle P moves so that, at time t, its displacement $\mathbf{r}$ from a fixed origin is given by

$$\mathbf{r} = \left(\mathrm{e}^t \cos t\right)\mathbf{i} + \left(\mathrm{e}^t \sin t\right)\mathbf{j}\,.$$

Show that the velocity of the particle always makes an angle of $\frac{\pi}{4}$ with the particle's displacement, and that the acceleration of the particle is always perpendicular to its displacement. Sketch the path of the particle for $0 \leqslant t \leqslant \pi$.

A second particle Q moves on the same path, passing through each point on the path a fixed time T after P does. Show that the distance between P and Q is proportional to e^t.

Discussion

The statement made in the 'preparation' section: 'Of course, to find the derivative of a vector, you just differentiate its components.' is not actually an 'of course' statement. You need to think about, for example.

$$\lim_{h\to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h}$$

which is the definition of the derivative. Then you write it out in terms of components to get the result stated above. You are not expected to do that here.

The result may be obvious, but does need justification; in fact, it would not be correct if the vector were written in polar coordinates and axes.

1993 S1 Q11

5 Preparation

Two equal thin rods AB and BC, each of length $2a$, are joined at B so that $\angle ABC = 90°$. Let G be the centre of gravity of the combined rods.

(i) Find the coordinates of G with respect to an origin at B, where A has coordinates $(0, 2a)$ and C has coordinates $(2a, 0)$.

(ii) Draw a diagram showing the rods.

Let X be the point on AB such that GX is perpendicular to AB. Find the values of $\tan BGX$ and $\tan AGX$.

(iii) On your diagram, draw the line through B that would be vertical if the rods were hung up by the point B.

On the same diagram, draw the line through A that would be vertical if the rods were hung up by the point A

Let α be the angle between these two lines. Show that $\tan\alpha = -2$.

6 The Mechanics question

A piece of uniform wire is bent into three sides of a square $ABCD$ so that the side AD is missing. Show that if it is first hung up by the point A and then by the point B then the angle between the two directions of BC is $\tan^{-1} 18$.

Discussion

Note how quickly a mechanics question can become a geometry question!

2006 S2 Q11

7 Preparation

A particle of mass m moves on a smooth horizontal surface. It experiences a constant force of magnitude F in the positive x-direction. Initially, the particle is at the origin and is moving with speed u in a direction that makes an angle of α with the positive x-axis ($\alpha = \frac{1}{2}\pi$ corresponds to the positive y-axis).

(i) Write down expressions for $\ddot{x}$ and $\ddot{y}$ and, by integration and using the initial conditions, obtain the position of the particle at time t.

(a) If $\alpha = -\pi$, find the time taken for the particle to return to the origin.
(b) If $\alpha = -\frac{3}{4}\pi$, find the time taken for the particle to reach the y-axis.
(c) If $\alpha = \frac{1}{2}\pi$, find the time taken for the particle to reach the line $y = x$.

(ii) Write $g \sin 2\theta + f \cos 2\theta$ in the form $R \sin(2\theta + \beta)$, and hence find its maximum value.

8 The Mechanics question

A projectile of unit mass is fired in a northerly direction from a point on a horizontal plain at speed u and an angle θ above the horizontal. It lands at a point A on the plain. In flight, the projectile experiences two forces: gravity, of magnitude g; and a horizontal force of constant magnitude f due to a wind blowing from North to South. Derive an expression, in terms of u, g, f and θ for the distance OA.

(i) Determine the angle α such that, for all $\theta > \alpha$, the wind starts to blow the projectile back towards O before it lands at A.

'Blow the projectile back towards O' means that the projectile's horizontal velocity is towards the origin.

(ii) An identical projectile, which experiences the same forces, is fired from O in a northerly direction at speed u and angle 45° above the horizontal and lands at a point B on the plain. Given that θ is chosen to maximise OA, show that

$$\frac{OB}{OA} = \frac{g - f}{\sqrt{g^2 + f^2} - f} \, .$$

Describe carefully the motion of the second projectile when $f = g$.

2008 S2 Q11

9 Preparation

(i) If $\tan\theta = \frac{3}{4}$ and θ is acute, find $\cos\theta$ and $\sin\theta$.

(ii) Here are some thoughts about solving this sort of mechanics problem.

- The first thing to do is to draw a BIG diagram. Draw the wedge with θ much less than 45° so that you can easily see which angles in the diagram are equal to θ and which are equal to $90 - \theta$.
- Put all the forces in your diagram. You may want to draw separate diagrams for the particle and the wedge, so you are not confused about which force acts on what.
- Remember Newton's third law. For example, if the particle experiences a frictional force, then the wedge experiences an equal and opposite frictional force.
- Remember that frictional forces oppose the motion. If you are not sure what direction a frictional force acts in, try to work out which direction the particle would move in if there were no friction.
- For static friction, the equation $F = \mu R$ holds only in limiting equilibrium, when the system is just about to move; but for kinematic friction, when the system is already in motion, $F = \mu R$ is assumed to hold.
- Write down the equations of motion (Newton's second law) for the various objects in the system. But before doing so, have a good look at the result you are aiming for, because it might help you to decide which directions to resolve the forces. For example, there is no g in the first displayed equation in the Mechanics question below: if you weren't sure whether to resolve forces horizontally and vertically, or parallel and perpendicular to the wedge, the absence of g in the equation is a pretty big clue.

10 The Mechanics question

A wedge of mass km has the shape (in cross-section) of a right-angled triangle. It stands on a smooth horizontal surface with one face vertical. The inclined face makes an angle θ with the horizontal surface. A particle P, of mass m, is placed on the inclined face and released from rest. The horizontal face of the wedge is smooth, but the inclined face is rough and the coefficient of friction between P and this face is μ.

(i) When P is released, it slides down the inclined plane at an acceleration a relative to the wedge. Show that the acceleration of the wedge is

$$\frac{a\cos\theta}{k+1}\,.$$

To a stationary observer, P appears to descend along a straight line inclined at an angle 45° to the horizontal. Show that

$$\tan\theta = \frac{k}{k+1}\,.$$

In the case $k = 3$, find an expression for a in terms of g and μ.

(ii) What happens when P is released if $\tan\theta \leqslant \mu$?

Discussion

If you obtained the first displayed equation by resolving forces horizontally for the particle and the wedge separately, you were probably pleased that the reactions and the frictional forces cancelled out. There is (of course) a reason for this. Even though there is friction in the system, so energy (KE+PE) is not conserved, the total momentum of the system is zero, because there are no external horizontal forces acting. Writing down the total horizontal momentum gives the displayed equation. The vertical momentum is not conserved because gravity and the reaction of the horizontal surface on the wedge are external forces.