STEP Support Programme

Assignment 11

Warm-up

1 **(i)** A function, T, is defined for positive integers k by

$$\mathrm{T}(k+1) = \mathrm{T}(k) + k + 1 \quad \text{and} \quad \mathrm{T}(1) = 1.$$

$T(2) = T(1) + 1 + 1$ etc.

Find (by calculating the values) the smallest value of n such that $T(n) > 100$.

(ii) The function f is defined for $0 \leqslant x \leqslant 2$ by

$$\mathrm{f}(x) = \begin{cases} x & \text{for} \quad 0 \leqslant x \leqslant 1 \\ (2-x)^2 & \text{for} \quad 1 < x \leqslant 2 \end{cases}$$

Sketch the graph of $\mathrm{f}(x)$ for $0 \leqslant x \leqslant 2$.

You are also given that $\mathrm{f}(x+2) = \mathrm{f}(x)$ for all x. Use this to find the values of $\mathrm{f}(2.5)$, $\mathrm{f}(3)$, $\mathrm{f}(3.5)$ and $\mathrm{f}(4)$.

Sketch the graph of $\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 6$.

Preparation

2 **(i)** By using the substitution $y = 2^x$, find the (real) value of x that satisfies the equation

$$4^x - 7 \times 2^x - 8 = 0$$

Note that the question says "value" and not "values", so you should expect just one solution.

(ii) Find the value of x that satisfies the equation

$$\sqrt{3x-5} - \sqrt{x+6} = 1$$

Remember that the notation $\sqrt{x}$ denotes the *positive* root of x, so $\sqrt{x} \geqslant 0$.

Discussion

It is a good idea to check your answers in the *original* equation, especially if you have been squaring things as this can introduce “extra” solutions. For example, the equation $x + 1 = 3$ has just one solution ($x = 2$), but if your first step had (rather bizarrely) been to square both sides to get $(x + 1)^2 = 9$ you would find two solutions ($x = 2$ and $x = -4$). Only one of these is a solution of the original equation.

The STEP question

3 **(i)** Use the substitution $\sqrt{x} = y$ (where $y \geqslant 0$) to find the real root of the equation

$$x + 3\sqrt{x} - \tfrac{1}{2} = 0\,.$$

(ii) Find all real roots of the following equations:

(a) $x + 10\sqrt{x+2} - 22 = 0\,;$

(b) $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0\,.$

Discussion

Making a substitution for the thing in the square root worked rather well for part (i); it might be worth trying it for part (ii).

Warm down

4 **(i)** Given that

$$2^{m+1} + 2^m = 3^{n+2} - 3^n$$

and that m and n are integers, find the values of m and n.

You could start by substituting some values of m and n to see what happens. If you chance upon a pair of values that work by doing this you should show that this is the only solution.

(ii) Find the values of x that satisfy the equation

$$3^{2x} - 34 \times 15^{x-1} + 5^{2x} = 0\,.$$

This one is a little trickier. One thing that might help is using two substitutions, one for 3^x and one for 5^x.