

## STEP Support Programme

### Assignment 11

#### Warm-up

- 1 (i) A function,  $T$ , is defined for positive integers  $k$  by

$$T(k+1) = T(k) + k + 1 \quad \text{and} \quad T(1) = 1.$$

$$T(2) = T(1) + 1 + 1 \text{ etc.}$$

Find (by calculating the values) the smallest value of  $n$  such that  $T(n) > 100$ .

- (ii) The function  $f$  is defined for  $0 \leq x \leq 2$  by

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ (2-x)^2 & \text{for } 1 < x \leq 2 \end{cases}$$

Sketch the graph of  $f(x)$  for  $0 \leq x \leq 2$ .

You are also given that  $f(x+2) = f(x)$  for all  $x$ . Use this to find the values of  $f(2.5)$ ,  $f(3)$ ,  $f(3.5)$  and  $f(4)$ .

Sketch the graph of  $f(x)$  for  $-2 \leq x \leq 6$ .

#### Preparation

- 2 (i) By using the substitution  $y = 2^x$ , find the (real) value of  $x$  that satisfies the equation

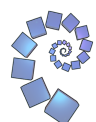
$$4^x - 7 \times 2^x - 8 = 0$$

Note that the question says “value” and not “values”, so you should expect just one solution.

- (ii) Find the value of  $x$  that satisfies the equation

$$\sqrt{3x-5} - \sqrt{x+6} = 1$$

Remember that the notation  $\sqrt{x}$  denotes the *positive* root of  $x$ , so  $\sqrt{x} \geq 0$ .



## Discussion

It is a good idea to check your answers in the *original* equation, especially if you have been squaring things as this can introduce “extra” solutions. For example, the equation  $x + 1 = 3$  has just one solution ( $x = 2$ ), but if your first step had (rather bizarrely) been to square both sides to get  $(x + 1)^2 = 9$  you would find two solutions ( $x = 2$  and  $x = -4$ ). Only one of these is a solution of the original equation.

## The STEP question

- 3 (i) Use the substitution  $\sqrt{x} = y$  (where  $y \geq 0$ ) to find the real root of the equation

$$x + 3\sqrt{x} - \frac{1}{2} = 0.$$

- (ii) Find all real roots of the following equations:

(a)  $x + 10\sqrt{x+2} - 22 = 0;$

(b)  $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0.$

## Discussion

Making a substitution for the thing in the square root worked rather well for part (i); it might be worth trying it for part (ii).

## Warm down

- 4 (i) Given that

$$2^{m+1} + 2^m = 3^{n+2} - 3^n$$

and that  $m$  and  $n$  are integers, find the values of  $m$  and  $n$ .

You could start by substituting some values of  $m$  and  $n$  to see what happens. If you chance upon a pair of values that work by doing this you should show that this is the only solution.

- (ii) Find the values of  $x$  that satisfy the equation

$$3^{2x} - 34 \times 15^{x-1} + 5^{2x} = 0.$$

This one is a little trickier. One thing that might help is using two substitutions, one for  $3^x$  and one for  $5^x$ .

