

## STEP Support Programme

### STEP 3 Calculus Questions

**1 2003 S3 Q1**

Given that  $x + a > 0$  and  $x + b > 0$ , and that  $b > a$ , show that

$$\frac{d}{dx} \arcsin \left( \frac{x+a}{x+b} \right) = \frac{\sqrt{b-a}}{(x+b)\sqrt{a+b+2x}}$$

and find  $\frac{d}{dx} \operatorname{arcosh} \left( \frac{x+b}{x+a} \right)$ .

Hence, or otherwise, integrate, for  $x > -1$ ,

(i)  $\int \frac{1}{(x+1)\sqrt{x+3}} dx$ ,

(ii)  $\int \frac{1}{(x+3)\sqrt{x+1}} dx$ .

[You may use the results  $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$  and  $\frac{d}{dx} \operatorname{arcosh} x = \frac{1}{\sqrt{x^2-1}}$ . ]

**2 2010 S3 Q2**

In this question,  $a$  is a positive constant.

(i) Express  $\cosh a$  in terms of exponentials.

By using partial fractions, prove that

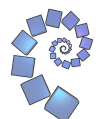
$$\int_0^1 \frac{1}{x^2 + 2x \cosh a + 1} dx = \frac{a}{2 \sinh a}.$$

(ii) Find, expressing your answers in terms of hyperbolic functions,

$$\int_1^\infty \frac{1}{x^2 + 2x \sinh a - 1} dx$$

and

$$\int_0^\infty \frac{1}{x^4 + 2x^2 \cosh a + 1} dx.$$



**3 2002 S3 Q1**

Find the area of the region between the curve  $y = \frac{\ln x}{x}$  and the  $x$ -axis, for  $1 \leq x \leq a$ . What happens to this area as  $a$  tends to infinity?

Find the volume of the solid obtained when the region between the curve  $y = \frac{\ln x}{x}$  and the  $x$ -axis, for  $1 \leq x \leq a$ , is rotated through  $2\pi$  radians about the  $x$ -axis. What happens to this volume as  $a$  tends to infinity?

**4 2004 S3 Q7**

For  $n = 1, 2, 3, \dots$ , let

$$I_n = \int_0^1 \frac{t^{n-1}}{(t+1)^n} dt.$$

By considering the greatest value taken by  $\frac{t}{t+1}$  for  $0 \leq t \leq 1$  show that  $I_{n+1} < \frac{1}{2}I_n$ .

Show also that  $I_{n+1} = -\frac{1}{n2^n} + I_n$ .

Deduce that  $I_n < \frac{1}{n2^{n-1}}$ .

Prove that

$$\ln 2 = \sum_{r=1}^n \frac{1}{r2^r} + I_{n+1}$$

and hence show that  $\frac{2}{3} < \ln 2 < \frac{17}{24}$ .

