STEP Support Programme

STEP 2 Calculus Topic Notes

Chain rule (function of a function)

$$\mathrm{h}'(x) = \mathrm{g}'\left(\mathrm{f}(x)\right)\mathrm{f}'(x) \quad \text{where} \quad \mathrm{h}(x) = \mathrm{g}\left(\mathrm{f}(x)\right) \quad \text{or}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x} \quad \text{where} \quad y = \mathrm{g}(u) \text{ and } u = \mathrm{f}(x)$$

Product rule

$$(\mathrm{f}(x)\mathrm{g}(x))' = \mathrm{f}'(x)\mathrm{g}(x) + \mathrm{f}(x)\mathrm{g}'(x) \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}(uv) = u \times \frac{\mathrm{d}v}{\mathrm{d}x} + v \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

Quotient rule

$$\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right)' = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2} \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{v \times \frac{\mathrm{d}u}{\mathrm{d}x} - u \times \frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

Implicit differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}(y)\right) = \mathrm{f}'(y)\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{e.g.} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(y^3\right) = 3y^2 \times \frac{\mathrm{d}y}{\mathrm{d}x}$$

Volume of revolution (about the x-axis)

$$V = \pi \int (\mathrm{f}(x))^2 \,\mathrm{d}x \quad \text{where} \quad y = \mathrm{f}(x)$$

Integration by parts

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Integration by substitution

$$\int \mathrm{f}(x)\,\mathrm{d}x = \int \mathrm{f}(\mathrm{g}(u)) \times \frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u \quad \text{where} \quad x = \mathrm{g}(u)$$

Separation of variables

If an equation can be written as $\mathrm{f}(y)\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(x)$ then we have

$$\int \mathrm{f}(y)\mathrm{d}y = \int \mathrm{g}(x)\mathrm{d}x$$

A very useful result

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

This is really just a special case of integration by substitution, but it is well worth knowing "off by heart"!

Top Tips!

Make sure you are familiar with the required formulae from the back of the STEP specification. It might be helpful to write them out and stick them on the back of the toilet door (or somewhere else)!

Integration

- Know your limits! When you substitute, remember to change variables in the limits too. For example $\int_1^a 1\,\mathrm{d}x$ under the substitution $u = 1/x$ becomes $\int_1^{1/a} (-1/u^2)\,\mathrm{d}u$.
- Use your limits! You may be able to look at how the limits change to indicate a likely substitution.
- You can swap the limits, but this introduces a minus sign; $\int_a^b = -\int_b^a$.
- $\int_0^{2\pi} \sin x \mathrm{d}x = 0$.
- $\int_0^{\pi} \sin^2 x \mathrm{d}x = \frac{\pi}{2}$.
- Four useful integration techniques:
  - Make a substitution,
  - Integrate by parts,
  - Express the function in partial fractions and use logarithms,
  - Guess and make it work (integration by inspection).

  This last one means guess what the answer is and then differentiate to see if you are correct, or in order to work out what multiple of your guess is correct.
- Remember signs when you're working with sines. SIN ≡ **S**ine **I**ntegrates **N**egatively. Alternatively, use "anti-differentiation", so if you know that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$, you know that $\int \cos x\,\mathrm{d}x = \sin x + k$.

Differential Equations

- Separation of variables, for $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$.
- Use a substitution to reach a solve-able differential equation (usually given to you — something like "Let $y = ux$"). Sometimes they will give you a substitution for the first differential equation and then leave you to work out what you need for the next one.
- Be careful with substitutions; check whether you're substituting for y or for x, and **work out the derivatives first!**