# STEP Support Programme

## STEP 2 Calculus Questions: Hints

**1** Remember to factorise, and don't divide by something which may be zero. Also, $\mathrm{e}^{-x^2} > 0$ so you can divide by this quite happily!

For the second part, start by differentiating $\mathrm{P}(x)\mathrm{e}^{-x^2}$. You can factorise out $\mathrm{e}^{-x^2}$ and then inside the bracket you need a polynomial with 5 solutions. If $\mathrm{Q}(x)$ has degree $n$ (so the highest power is $x^n$) then there are at most $n$ solutions to $\mathrm{Q}(x) = 0$. Also, if $\mathrm{Q}(x)$ has degree $n$ then $x\mathrm{Q}(x)$ has degree $n + 1$.

You are asked to find "a polynomial", which means any polynomial that works will do, so it makes sense to go for the simplest one.

**2** Note that $\sin(\pi - t) = \sin t$ (draw a picture to convince yourself). Also you can use a substitution of $t = x$ (which basically changes the letter used as the variable inside the integral).

We have $\int \mathrm{g}(x)\mathrm{d}x + \int \mathrm{h}(x)\mathrm{d}x = \int (\mathrm{g}(x) + \mathrm{h}(x))\,\mathrm{d}x$, and if $I + I = A$, then $I = \frac{1}{2}A$.

**(i)** Here we can apply the result found in the "stem" of the question, taking $\mathrm{f}(\sin x) = \dfrac{\sin x}{3 + \sin^2 x}$. Now use the substitution $u = \cos x$, and the resulting integral can be evaluated by another substitution or partial fractions.

Note that $\displaystyle\int \frac{1}{A + kx}\mathrm{d}x = \frac{1}{k}\ln|A + kx|$.

**(ii)** You cannot just use the result shown in the stem here as the limits are different. Start by writing the integral as:

$$\int_0^{\pi} x\mathrm{f}(x)\mathrm{d}x + \int_{\pi}^{2\pi} x\mathrm{f}(x)\mathrm{d}x\,.$$

Then look for a substitution which will make the limits in the second integral the same as the limits in the first integral.

**(iii)** We have $|\sin 2x| = |2\sin x \cos x|$, and as $\sin x \geqslant 0$ in this range we can write this as $2\sin x|\cos x|$. From $\sin^2 x + \cos^2 x = 1$ we get $|\cos x| = \sqrt{1 - \sin^2 x}$ (and hence it has the form required for the stem result).

**3** **(i)** The first part needs the chain rule (repeatedly) whilst the second needs both the chain rule and the product rule. The request to simplify is to help you answer the next bit!

When a question says "Hence", it is perfectly within the examiner's rights to give you no marks for trying to solve the question in a different way (unlike "hence, or otherwise"). Try and combine your two previous answers to help you answer this one.

Note that $\sqrt{3+x^2} = \dfrac{3+x^2}{\sqrt{3+x^2}}$. You should be able to write this in terms of your previous answers.

**(ii)** You can treat this differential equation as a quadratic in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find two simpler differential equations. Your final answer to part **(i)** should be helpful.

**4** **(i)** Using the given substitution and careful manipulation should lead you to a differential equation where the variables are separable. Don't forget to substitute back for $y$ again at the end.

**(ii)** Try to use a similar idea as for part **(i)**. Look at how this equation is different to help you decide which substitution to use.

**(iii)** This part is asking you to look at your previous two results and the generalise. Simplifying your answers to the previous two parts may help you to spot the general pattern.

**5** For the first part, substitute in $y = \mathrm{e}^x$ and show that this does satisfy the equation $(*)$. Then substitute $y = u\mathrm{e}^x$ (remember that $u$ is a function of $x$ so you will need the product rule to differentiate $u\mathrm{e}^x$).

Even if you don't quite get to $(**)$ you can still attempt the next part of the question — you should get a first order differential equation for $v$ in which the variables are separable, and having found $v$ you can then use $\dfrac{\mathrm{d}u}{\mathrm{d}x} = v$ to find $u$. The final request is a "hence", so this means that you must use your previous results when answering this part. In particular, substituting $y = Ax + B\mathrm{e}^x$ into $(*)$ will not gain any credit.