# STEP Support Programme

## STEP 3 Polar Coordinates and other Coordinate Geometry: Hints

**1** Start by drawing a good clear diagram. Read the whole question before you draw your diagram, as there is information later in the question about the positions of $S, T, U$ and $V$ that you might wish to include in your diagram.

The first part of the question is basically finding the equation of straight lines and solving simultaneous equations. Once you have a quadratic equation for the second part, remember that $s$ and $t$ are the $x$ coordinates of the roots of a quadratic in $x$, so you can write $(x-s)(x-t)=0$.

Finally, express $p+q$ using your expressions for $p$ and $q$, find expressions for $u+v$ and $uv$, and use these together with your expressions for $s+t$ and $st$ to show that $p+q=0$.

**2** Begin by drawing one or more diagrams to make sense of the information in the question. In particular, take time to see what each circle has in common, and draw in any useful lines to give similar right-angled triangles as you are asked for a trigonometric relationship for each circle. Don't be afraid to introduce extra notation so that you can clearly reference the points you are using in your answer.

For circles $C_1, C_2, C_3$... the areas form a geometric series so you can calculate the sum to infinity, and then add on the area of the semicircle $C_0$.

By finding the area of the triangle $T$ and differentiating $\frac{S}{T}$ with respect to $\alpha$, you can find the value of $\cos\alpha$ at the maximum and deduce that $\frac{S}{T} > \frac{4}{5}$ here.

**3** At a point of inflection we have $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$, so start by finding the second derivative and setting it equal to 0. The existence of a point with $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ does not mean that that point is a point of inflection, but if there are no points where $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ then there are no points of inflection.

Use $x=r\cos\theta$ and $y=r\sin\theta$ in $x^3+y^3=3xy$ to find $r^2$ in terms of $\theta$. You will then have an integral in $\theta$ to evaluate. Start by dividing throughout by $\cos^6\theta$, and then you will need to make two substitutions.

**4** Find the gradient of the tangent at $P$ by differentiating, and use this to get the equation of $ON$. Then find the equation of $SP$, and use these equations to find the $y$ coordinate of their intersection.

For the second part of the question, start by finding the $x$ coordinate of $T$. The equation of the given circle is $(x+ea)^2+y^2=a^2$; start with the LHS and substitute in for $x$ and $y$ at $T$, and after quite a lot of algebra you should get $a^2$.

**5** Don't be put off by the length and wordiness of this question. If you read each step carefully and follow the instructions, it's a lot easier than it looks!

You are told to begin with the expression $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ so you'll need expressions for $r$ and $\theta$ in terms of $x$ and $y$. When you differentiate $\tan\theta$ you get $\sec^2\theta$ which rather handily can be written in terms of $\tan\theta$ allowing you to rearrange, giving you an expression for $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ in terms of $x$ and $y$ to substitute into $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ to get (*).

The next very wordy section is defining terms for the rest of the question. Note that $P$ is defined consistently with the first part of the question, so $[P]$ is exactly $(*)$. Substitute your coordinates for $A$ (which will be of the form $X=\mathrm{f}(x)$ and $Y=\mathrm{g}(y)$) into $(*)$ and rearrange, being careful with minus signs. The expression for $[B]$ can be deduced by comparing the coordinates of $A$ and $B$, and as $[A]=[B]$ the last part can be deduced.