STEP Support Programme

STEP 2 Equations Questions: Hints

1 This question does not ask you to sketch any graphs, but it might be a very good idea to!

(i) You could sketch the curve $y = (x-1)^4 + (x+1)^4$ and then consider how many times it would intersect $y = c$ for different values of c. You will have to carefully justify the number of turning points. Perhaps a clearer method is to consider a quadratic equation.

(ii) Again, a sketch could work. Or you might be able to relate this to part **(i)**.

(iii) Definitely a sketch here! Your graph should consist of three straight lines. Remember that:

$$|x-a| = \begin{cases} x-a & \text{for } x > a \\ a-x & \text{for } x < a\,. \end{cases}$$

(iv) Another sketch. Remember that cubics must have at least one root.

2 For the first part, $\mathrm{e} = \mathrm{e}^1$. Show that $4! > 2^4$ and then consider what each side will be multiplied by to get $5!$ and 2^5 etc. (you can use proof by induction if you wish).

You don't need to find the coordinates of the minimum, you can look at the sign of the gradient for $x = \frac{1}{2}$ and $x = 1$. You can also think about what happens to the gradient as $x \to \frac{4}{3}$ (from below, as the curve is undefined for $x \geqslant \frac{4}{3}$).

3 **(i)** This is an "if and only if" so be careful! Start by trying to solve for y and z using the first two equations, and then if all three have a solution then $b = \cdots$. Then go the other way — i.e. show that if $b = 11$ then the equations have a solution.

(ii) If you try to eliminate a variable (probably y or z) you will find that you end up with the same equation twice. Instead, try setting $z = \lambda$ and then use two equations to find x and y in terms of a and λ. You must check the third equation!

(iii) Use your previous answer with $a = 2$ to write $x^2 + y^2 + z^2$ in terms of λ. You can then minimise this (Calculus is perhaps not the most efficient method here).

(iv) Use the general solution from part **(ii)**. The condition $y^2 + z^2 < 1$ will give a condition on λ.

4 **(i)** Find the coordinates of the turning points. Then show whether they are above or below the x-axis. Remember that something squared is always greater than or equal to zero.

(ii) You can use the quadratic formula to find the two possible values of u^3. These both give the same value of x (which is to be expected as there is only one real root of x).

(iii) Use $t^2 - pt + q \equiv (t - \alpha)(t - \beta)$ and it will be helpful to consider $(\alpha + \beta)^3$. The condition on the roots means that either $\alpha^2 = \beta$ or $\beta^2 = \alpha$. There is a connection back to the first two parts of the question.