STEP Support Programme

STEP 2 Mechanics Questions: Hints

1 **(i)** One strategy is to start by dividing the trapezium into a rectangle and triangle. The centre of mass at the rectangle will be where the two diagonals meet and the centre of mass of the triangle is where the three medians of the triangle meet. You can then either consider that there is zero resultant moment about the centre of mass, or use the weighted mean x coordinate of the two centres of mass. Note that the rectangle has double the mass of the triangle.

(ii) Start by thinking of the tank as a combination of 5 different faces. You can locate the x coordinate of each face. You will also need the mass of each face — it may be helpful to have a symbol such as ρ for the mass per cm^2. As in the first part, you can find the weighted mean of the x coordinates (i.e. the horizontal distance away from the back of the tank, or you can consider moments.

For the last part you are given that $d = 20$, so you can find the area of the metal used to form the tank and the volume of the liquid. This will enable you to find the mass of both the tank and the liquid (this will be in terms of k and maybe something else, such as ρ). You can then use one of the methods used earlier again.

2 Start by drawing a large, clear diagram including all the forces acting on the rod (the weight of the rod, the tension in the string and the reaction force). You can then resolve horizontally and vertically, as well as taking moments (taking moments about point A is probably best). This will give you three equations to work with, and their is quite a lot of trigonometrical messing about to be done.

It may be helpful to find an exact value for $\tan 15°$.

3 **(i)** 3D projectile questions can be tricky. Start by drawing a diagram showing the locations of O, B and the direction of travel of the particle. The only force acting on the particle is gravity, so SUVAT can be used.

If we let θ be the angle above the horizontal that the particle is being projected at, then we have $\tan\theta = \frac{1}{2}$. You can use this to find the value of $\cos\theta$ and $\sin\theta$.

When you have the position vector of the particle the magnitude of this will give the distance of the particle from O. You can find the value of t which minimises this in order to find P.

(ii) This part is considerably shorter to do than the first part! You can maximise the $\mathbf{k}$ component of the position of the particle, and then show that at this time the particle is at P.

(iii) The bullet can be considered as going in a straight line from O to P. Start by working out the distance OP, and then you can work out how long the bullet takes to reach P. The particle will have moved a bit further at this time, and you should be able to work out how far it has travelled.

4 Start by deciding on what to call all the different velocities. What you call them does not matter, as long as you can remember which is which. It is helpful to write down a list of what the different symbols mean.

You also need to decided which direction is going to be considered "positive".

Do check that your answers make sense, and that one particle has not "passed through" the other.

For the last part you might want to consider the sign of $1 - 4e^2$ for different values of e.

5 It may be helpful to separate out the movement of P around the hoop and the horizontal movement of the whole hoop. You could so this by considering the position of the centre of the hoop as it rolls, and also by thinking about the position of a particle moving round a stationary hoop.

The speed of one of the particles is given by $\sqrt{\dot{x}^2 + \dot{y}^2}$.

Given the only forces are gravity and the reaction force between the hoop and the table, the energy of the hoop is conserved. The total energy for a system of particles is the sum of the kinetic energy and potential energy for each particle (the hoop is "light" so we consider it to have zero mass.).

6 Start by deciding which direction is positive, and what you are going to call the different velocities of the particles. It may be helpful to write down a list, or a series of diagrams showing the different velocities before and after each collision.

For the two collisions there will be a total of 4 equations (Newton's Experimental Law and conservation of momentum). Use these to write e' in terms of the masses. We know that $0 \leqslant e' \leqslant 1$.

You can find an expression for the final energy of the system just in terms of the masses. The only thing that can vary is m_2, so it may be helpful to re-write the condition of the masses previously shown in the form $\cdots \leqslant m_2 \leqslant \cdots$.

7 Start by considering the forces on the bottom string (string 1), which will just be the force due to the weight of the mass on the bottom and the tension in the string. The consider the situation with the "r^{th}" string and write down an equation for the forces involved. You might like to call the tension in the "r^{th}" string T_r. You should be able to find T_r in terms of T_{r-1} and hence use the value of T_1 to find an explicit formula for T_r.

You can use Hooke's law to find the extension in each short string, and then use a sum to find the length of the long string. You can do something similar for the elastic potential energy. The formulae for $\sum_{i=1}^{n} r$ and $\sum_{i=1}^{n} r^2$ will be useful.

For the uniform heavy rope, you can think of it as being made up of lots of little light stings with weights on the end, i.e. consider what happens as n gets really large.