STEP Support Programme

STEP 3 Mechanics: Hints

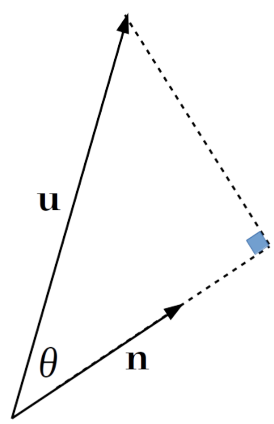
In a lot of these questions it is helpful to start by thinking about what is happening physically in the question — to "get a sense" of what the question is about. For example, in question **(5)** you would expect the sphere to float in the liquid, and if pushed down it will "bob" up and down a bit until it comes back to rest.

1 Since the collision is "perfectly elastic" we know that $e = 1$.

The conservation of momentum equation can be written in vector form (which will be easier than trying to consider components!).

For oblique collisions Newtons experimental law holds in the direction of the line connecting the centres of the discs. This line will be in the same direction as the direction of the impulse on the second disc, and since the second disc starts at rest and ends up moving in direction $\mathbf{n}$, the line connecting the centres of the discs has direction $\mathbf{n}$.

The component of velocity of the first disc acting in the direction of $\mathbf{n}$ is equal to $\mathbf{u} \cdot \mathbf{n}$. This is shown in the diagram below:



The component of velocity in the same direction as $\mathbf{n}$ is equal to $|\mathbf{u}| \cos\theta$ and we also have $\mathbf{u} \cdot \mathbf{n} = |\mathbf{u}||\mathbf{n}| \cos\theta = |\mathbf{u}| \cos\theta$ as $|\mathbf{n}| = 1$.

Using the two equations you can then eliminate the velocity of the first disc after the collision and then obtain the required result.

For part **(ii)** equate the kinetic energies and eliminate the velocity of the first disc after the collision. Use the result from part **(i)** to eliminate the velocity of the second disc after the collision and remember that $\mathbf{u} \cdot \mathbf{n} = |\mathbf{u}||\mathbf{n}| \cos\theta$. It will be useful to note that $0 \leqslant \cos^2\theta \leqslant 1$.

2 Start by having a think about what is happens when the spring is released, and what different things could happen depending on how much compression the spring is under. These will help you with the different cases.

Draw a diagram!

The given energy started in the spring tells you something about the natural length of the spring.

Energy is conserved, which will help you find an equation for θ. Differentiating this equation is helpful.

There will be a different kind of motion for $\beta > \gamma$ and $\beta < \gamma$ (for some angle γ). Something else will happen when $\beta = \gamma$.

3 You may have seen a hamster running in a wheel before, where the axis about which the wheel rotates is at the centre of the wheel (O). Here the axis is on the rim of the "wheel", so that the wheel can "swing".

Draw a diagram showing the hoop and the mouse. It may be helpful to show the forces acting on the hoop and those acting on the mouse in different colours.

In equilibrium the hoop will not turn (so no net moment) and the only acceleration the mouse experiences is the one radially into the centre of the circle. Find some equations and use them to eliminate lots of things. You should be able to find u in terms of a and g.

Thinking about the problem physically, you should be able to predict what will happen to the hoop as the mouse goes further "up the side". Limiting friction is a useful thing to consider.

4 First draw a diagram, and decide where Potential Energy is going to be zero (bottom of the hoop would seem to be a good place). Use the point at which the string becomes slack and the initial position, and the fact that energy is conserved (no forces such as friction acting) to find the modulus of elasticity.

If the string is making an angle ϕ with the downward vertical, you can write the angle the radius to the ring makes with the vertical in terms of ϕ. Use conservation of energy and "$F = ma$" radially to write R in terms of $\cos\phi$.

Note that the question says "the magnitude of R".

5 You can use a "Volume of revolution" integral to find the required volume. It would be better not to use "x" as one of the axes as this has already been defined in the question.

Use "$F = ma$" to write down a differential equation for x. If you don't get the given equation it may be that your sign for $\ddot{x}$ is incorrect (in which case try changing it and see if that works!).

When considering the small oscillations it will be helpful to write "$x = \frac{1}{2}R + \epsilon$" or similar where ϵ is "small" (more precisely $\left|\frac{\epsilon}{R}\right| << 1$).