STEP Support Programme

STEP 3 Mechanics Questions

1 2000 S3 Q9

Two small discs of masses m and μm lie on a smooth horizontal surface. The disc of mass μm is at rest, and the disc of mass m is projected towards it with velocity $\mathbf{u}$. After the collision, the disc of mass μm moves in the direction given by unit vector $\mathbf{n}$. The collision is perfectly elastic.

(i) Show that the speed of the disc of mass μm after the collision is $\dfrac{2\mathbf{u}\cdot\mathbf{n}}{1+\mu}$.

(ii) Given that the two discs have equal kinetic energy after the collision, find an expression for the cosine of the angle between $\mathbf{n}$ and $\mathbf{u}$ and show that $3-\sqrt{8} \leqslant \mu \leqslant 3+\sqrt{8}$.

2 2007 S3 Q9

Two small beads, A and B, each of mass m, are threaded on a smooth horizontal circular hoop of radius a and centre O. The angle θ is the acute angle determined by $2\theta = \angle AOB$.

The beads are connected by a light straight spring. The energy stored in the spring is

$$mk^2a^2(\theta-\alpha)^2,$$

where k and α are constants satisfying $k > 0$ and $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$.

The spring is held in compression with $\theta = \beta$ and then released. Find the period of oscillations in the two cases that arise according to the value of β and state the value of β for which oscillations do not occur.

3 2004 S3 Q9

A circular hoop of radius a is free to rotate about a fixed horizontal axis passing through a point P on its circumference. The plane of the hoop is perpendicular to this axis. The hoop hangs in equilibrium with its centre, O, vertically below P. The point A on the hoop is vertically below O, so that POA is a diameter of the hoop.

A mouse M runs at constant speed u round the rough inner surface of the lower part of the hoop. Show that the mouse can choose its speed so that the hoop remains in equilibrium with diameter POA vertical.

Describe what happens to the hoop when the mouse passes the point at which angle $AOM = 2\arctan\mu$, where μ is the coefficient of friction between mouse and hoop.

4 2012 S3 Q10

A small ring of mass m is free to slide without friction on a hoop of radius a. The hoop is fixed in a vertical plane. The ring is connected by a light elastic string of natural length a to the highest point of the hoop. The ring is initially at rest at the lowest point of the hoop and is then slightly displaced. In the subsequent motion the angle of the string to the downward vertical is ϕ. Given that the ring first comes to rest just as the string becomes slack, find an expression for the modulus of elasticity of the string in terms of m and g.

Show that, throughout the motion, the magnitude R of the reaction between the ring and the hoop is given by

$$R = (12\cos^2\phi - 15\cos\phi + 5)mg$$

and that R is non-zero throughout the motion.

5 2013 S3 Q9

A sphere of radius R and uniform density ρ_s is floating in a large tank of liquid of uniform density ρ. Given that the centre of the sphere is a distance x above the level of the liquid, where $x < R$, show that the volume of liquid displaced is

$$\frac{\pi}{3}(2R^3 - 3R^2x + x^3)\,.$$

The sphere is acted upon by two forces only: its weight and an upward force equal in magnitude to the weight of the liquid it has displaced. Show that

$$4R^3\rho_\text{s}(g + \ddot{x}) = (2R^3 - 3R^2x + x^3)\rho g\,.$$

Given that the sphere is in equilibrium when $x = \frac{1}{2}R$, find ρ_s in terms of ρ. Find, in terms of R and g, the period of small oscillations about this equilibrium position.