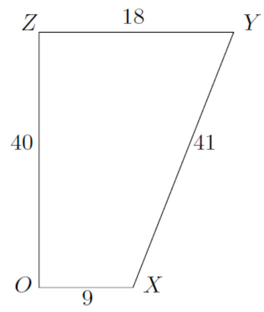
STEP Support Programme

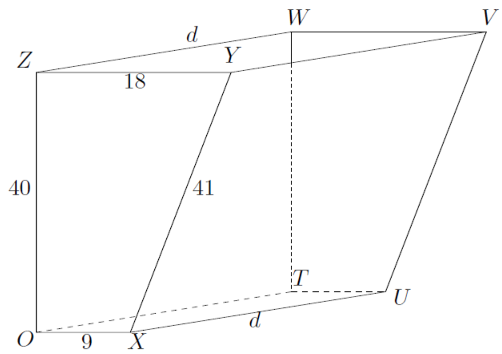
STEP II Mechanics Questions

1 **2009 S2 Q9**

(i) A uniform lamina $OXYZ$ is in the shape of the trapezium shown in the diagram. It is right-angled at O and Z, and OX is parallel to YZ. The lengths of the sides are given by $OX = 9\,\text{cm}$, $XY = 41\,\text{cm}$, $YZ = 18\,\text{cm}$ and $ZO = 40\,\text{cm}$. Show that its centre of mass is a distance $7\,\text{cm}$ from the edge OZ.



(ii) The diagram shows a tank with no lid made of thin sheet metal. The base $OXUT$, the back $OTWZ$ and the front $XUVY$ are rectangular, and each end is a trapezium as in part **(i)** . The width of the tank is $d\,$cm.



Show that the centre of mass of the tank, when empty, is a distance

$$\frac{3(140 + 11d)}{5(12 + d)}\ \text{cm}$$

from the back of the tank.

The tank is then filled with a liquid. The mass per unit volume of this liquid is k times the mass per unit area of the sheet metal. In the case $d = 20$, find an expression for the distance of the centre of mass of the filled tank from the back of the tank.

2 **2010 S2 Q11**

A uniform rod AB of length $4L$ and weight W is inclined at an angle θ to the horizontal. Its lower end A rests on a fixed support and the rod is held in equilibrium by a string attached to the rod at a point C which is $3L$ from A. The reaction of the support on the rod acts in a direction α to AC and the string is inclined at an angle β to CA. Show that

$$\cot\alpha = 3\tan\theta + 2\cot\beta\,.$$

Given that $\theta = 30^\circ$ and $\beta = 45^\circ$, show that $\alpha = 15^\circ$.

3 **2007 S2 Q11**

In this question take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$ *and neglect air resistance.*

The point O lies in a horizontal field. The point B lies $50\,\mathrm{m}$ east of O. A particle is projected from B at speed $25\,\mathrm{m\,s^{-1}}$ at an angle $\arctan\frac{1}{2}$ above the horizontal and in a direction that makes an angle 60° with OB; it passes to the north of O.

(i) Taking unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ in the directions east, north and vertically upwards, respectively, find the position vector of the particle relative to O at time t seconds after the particle was projected, and show that its distance from O is

$$5(t^2 - \sqrt{5}t + 10)\,\mathrm{m}.$$

When this distance is shortest, the particle is at point P. Find the position vector of P and its horizontal bearing from O.

(ii) Show that the particle reaches its maximum height at P.

(iii) When the particle is at P, a marksman fires a bullet from O directly at P. The initial speed of the bullet is $350\,\mathrm{m\,s^{-1}}$. Ignoring the effect of gravity on the bullet show that, when it passes through P, the distance between P and the particle is approximately $3\,\mathrm{m}$.

4 **2011 S2 Q9**

Two particles, A of mass $2m$ and B of mass m, are moving towards each other in a straight line on a smooth horizontal plane, with speeds $2u$ and u respectively. They collide directly. Given that the coefficient of restitution between the particles is e, where $0 < e \leqslant 1$, determine the speeds of the particles after the collision.

After the collision, B collides directly with a smooth vertical wall, rebounding and then colliding directly with A for a second time. The coefficient of restitution between B and the wall is f, where $0 < f \leqslant 1$. Show that the velocity of B after its second collision with A is

$$\tfrac{2}{3}(1-e^2)u - \tfrac{1}{3}(1-4e^2)fu$$

towards the wall and that B moves towards (not away from) the wall for all values of e and f.