STEP Support Programme

STEP 2 Mechanics Topic Notes

- **SUVAT equations for CONSTANT acceleration**

  In the following equations, s = displacement, u = initial velocity, v = final velocity, a = (constant) acceleration and t = time.
  You will need to know these, or be able to derive them all very quickly! Velocity, acceleration and displacement are all vector quantities.

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \frac{(u+v)}{2}t$$

- **Non-constant acceleration**

  When acceleration is not constant you can use calculus, e.g.

$$v = \frac{\mathrm{d}x}{\mathrm{d}t}, \quad a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}, \quad v = \int a\,\mathrm{d}t, \quad x = \int v\,\mathrm{d}t$$

  This idea can be extended to 2 dimensions (and above) by using vectors.

- **Newton's Laws of Motion**

  N1: An object will remain at rest or continue to move in a straight line at constant speed unless it is acted on by a resultant force
  N2: $F = ma$, where F is the resultant force, m is the mass of the object, and a is its acceleration.
  N3: Every action has an equal and opposite reaction.

- **Friction**

  Friction always acts to oppose the direction the object is trying to move in (so a particle placed at rest on a slope is trying to move down, so friction acts up the slope, whereas a particle attached to a string being pulled up a slope has friction acting down the slope). Friction can increase up to a maximum value (given by $F = \mu R$, after which it cannot be increased and so the particle can move).

- **Collisions**

  Conservation of Momentum: Total momentum before an impact = Total momentum after an impact.
  If a Force F acts for a time t then Impulse = Ft, and Impulse = Change in momentum.
  Remember: Momentum is a Vector quantity as it is the product of mass and velocity!
  Newton's Experimental Law:

  $$\text{speed of separation of particles} = e \times \text{speed of approach of particles}$$

  where e is the coefficient of restitution.
  $0 \leq e \leq 1$. When $e = 0$, the collision is *inelastic* and particles coalesce (stick together). When $e = 1$, the collision is *perfectly elastic*, and there is no loss of kinetic energy. I find that I make fewer sign errors if I use this form of the law rather than u_1, v_2 etc.

- **Elastic strings and springs**

  Hooke's Law $T = kx = \dfrac{\lambda x}{l}$ where T is the tension, x is the extension, k is stiffness, λ is the modulus of elasticity and l is the natural length of the string/spring. If a spring is in compression, x will be the amount the spring is compressed by and T is the "Thrust".

- **Energy**

  Kinetic energy of a particle: $\frac{1}{2}mv^2$
  Kinetic energy of a particle in 2-dimensions: $\frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right)$
  Potential energy: mgh where h is the height above some reference point (which can be chosen to make the problem easier)
  Elastic Potential energy: the EP_E stored in an elastic string or spring is $\dfrac{\lambda x^2}{2l}$

  The **total energy** of an **isolated system** is conserved. You can usually ignore energy which is dissipated as heat, so $K_E + P_E + EP_E$ is a constant.

  The **Work Done** on a system by external forces (excluding weight) is equal to the total change in the mechanical energy (K.E., P.E. and E.P.E.) of the system.

  The **power** is the amount of energy transferred per unit time. We have Power $= \frac{\text{change in energy}}{\text{time}}$.
  Power as a function of time is $P = \dfrac{\mathrm{d}W}{\mathrm{d}t}$.

- **Projectiles**

  A projectile's motion can be considered in terms of its horizontal and vertical component. Consider a particle fired at an angle θ above the horizontal, with initial speed u, where the only force acting is gravity.
  Horizontally, it travels at constant speed governed by

  $$s_x = u\cos\theta t$$

  Vertically, it travels with constant negative acceleration due to gravity, governed by

  $$s_y = u\sin\theta t - \frac{1}{2}gt^2$$

- **Moments and Centre of Mass**
  The moment of a force about a point is found by multiplying the magnitude of the force by the perpendicular distance from the point to the line of action of the force. The *Centre of Mass* of a body is the point at which the mass can be assumed to be concentrated, for the purposes of mechanically modelling a situation.
  The position of the centre of mass of any shapes used will either be given or deducible by symmetry. The centre of mass of a compound shape can be found by using moments — the moments about the Centre of Mass will be zero. Alternatively you can find the weighted mean of the x-coordinates and y-coordinates of the individual centres of mass. .

  When pushing an object to try and make it move it may "topple" or "tip" before it "slides" or "slips". To work out which happens first you can work out the minimum force you need to apply to overcome friction and make the object "slip" (by resolving forces in the direction the object will slip in). Then work out the minimum force need to topple/tip the object by considering moments (usually taken about the "front edge" of the object, i.e. the edge the object will tip about if it does tip). You can then find which force is smaller and hence whether the object tips or slips first.

Top Tips!

- Choose which direction is positive, and STICK TO IT! It is usual to choose up and right as positive directions, and down and left as negative. As long as you are consistent within a question you can choose differently, but it is easy to make a mistake about direction of travel if you are not clear.

- Year after year, the examiners' reports comment on candidates' unwillingness to draw good, clear diagrams. Begin any mechanics question by drawing a large diagram and marking on all the information you know. Make directions of motion or force action very clear using arrows. Don't be afraid to annotate your diagram as you go along or draw a new one, to make the situation clear to yourself as well as to the examiner.

- When resolving forces, some people find it easier to use a different colour, or pencil instead of pen, so they can see the original force along with its components.

- Many STEP Mechanics questions consist of a little mathematical modelling followed by a LOT of algebra and trigonometry. Don't be put off if there are a lot of dense equations to keep track of - you may be used to questions where you are given numbers rather than letters to represent speeds, masses, forces etc, so it can seem overwhelming when you have six different letters at once. It will come with practice, but if you need to do a 'sanity check' put some numbers in!

- As with all STEP questions, don't get hung up on using particular letters for particular things. u and v are usually initial and final velocities, but if they're used to represent something else in the question, use capitals, or other letters, or subscripts. Whenever you introduce a new variable, make it clear what it is; the examiner can probably guess what u_a, u_b, v_a, v_b, m_a and m_b are, but better to keep everything clearly defined! It's easy in the dense algebra to mistake an m for an M - don't be that person!