STEP Support Programme

STEP 2 Curve Sketching Questions: Hints

1 Start by differentiating (product and chain rule needed). To simplify the result it is easier to take out a common factor of $(x-2)^3$ first rather than expanding everything and then trying to factorise a quintic.

There are three stationary points — two of these can be classified by using the second derivative. The nature of the third one can be deduced by looking at the intersections with the axes, the nature of the first two stationary points and the behaviour as $x \to \pm\infty$.

For parts **(i)** and **(ii)** it is probably a good idea to draw the original curve first (either faintly or dotted etc. so that it is clear which is your actual answer).

2 **(i)** When you differentiate you get a quadratic, and you are told that the roots of this are $x = p$ and $x = q$. This means that the derivative can be written as $k(x-p)(x-q)$.

(ii) Use the facts $p > 0$ and $n > 0$ to approximately place the turning points.

(iii) m is the y coordinate when $x = p$, so $m = 2p^3 - bp^2 + cp$. You will need to use factorisation and your results from part **(i)**.

(iv) Make sure you have drawn $x = p$ and $y = n$ on your sketch. A couple more lines might be useful.

3 **(i)** Probably the best starting point is to work out what the domain of x is (i.e. the values of x that are actually possible). There is one stationary point. Sketch another line on your graph to show that there is one value of x that satisfies the given equation. You should be able to give the solution just from looking at your sketch (though it would be a good idea to check that it is a solution!).

(ii) Here sketch two curves, and use this to see how many solution are possible. Some algebraic manipulation will be needed to find the solution(s).

4 The derivative looks a bit messy at first, but will simplify quite nicely. If $a > 1$ think about what you know about $y = x^2 - 2x + a$. For the sketches it will be helpful to note that when x is "large", $y \approx \frac{x}{\sqrt{x^2}}$. Note that $\sqrt{x^2}$ is not always equal to x.