

## STEP Support Programme

## STEP 2 Curve Sketching Questions

**1 2004 S2 Q3**

The curve  $C$  has equation

$$y = x(x + 1)(x - 2)^4.$$

Determine the coordinates of all the stationary points of  $C$  and the nature of each. Sketch  $C$ .

In separate diagrams draw sketches of the curves whose equations are:

(i)  $y^2 = x(x + 1)(x - 2)^4$ ;

(ii)  $y = x^2(x^2 + 1)(x^2 - 2)^4$ .

**2 2007 S2 Q2**

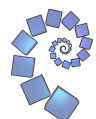
A curve has equation  $y = 2x^3 - bx^2 + cx$ . It has a maximum point at  $(p, m)$  and a minimum point at  $(q, n)$  where  $p > 0$  and  $n > 0$ . Let  $R$  be the region enclosed by the curve, the line  $x = p$  and the line  $y = n$ .

(i) Express  $b$  and  $c$  in terms of  $p$  and  $q$ .

(ii) Sketch the curve. Mark on your sketch the point of inflection and shade the region  $R$ . Describe the symmetry of the curve.

(iii) Show that  $m - n = (q - p)^3$ .

(iv) Show that the area of  $R$  is  $\frac{1}{2}(q - p)^4$ .



**3 2011 S2 Q1**

- (i) Sketch the curve  $y = \sqrt{1-x} + \sqrt{3+x}$ .

Use your sketch to show that only one real value of  $x$  satisfies

$$\sqrt{1-x} + \sqrt{3+x} = x + 1,$$

and give this value.

- (ii) Determine graphically the number of real values of  $x$  that satisfy

$$2\sqrt{1-x} = \sqrt{3+x} + \sqrt{3-x}.$$

Solve this equation.

**4 1999 S2 Q7**

The curve  $C$  has equation

$$y = \frac{x}{\sqrt{x^2 - 2x + a}},$$

where the square root is positive. Show that, if  $a > 1$ , then  $C$  has exactly one stationary point.

Sketch  $C$  when (i)  $a = 2$  and (ii)  $a = 1$ .

