STEP Support Programme

STEP 2 Curve Sketching Questions

1 2004 S2 Q3

The curve C has equation

$$y = x(x+1)(x-2)^4.$$

Determine the coordinates of all the stationary points of C and the nature of each. Sketch C.

In separate diagrams draw sketches of the curves whose equations are:

(i) $y^2 = x(x+1)(x-2)^4$;

(ii) $y = x^2(x^2+1)(x^2-2)^4$.

2 2007 S2 Q2

A curve has equation $y = 2x^3 - bx^2 + cx$. It has a maximum point at (p, m) and a minimum point at (q, n) where $p > 0$ and $n > 0$. Let R be the region enclosed by the curve, the line $x = p$ and the line $y = n$.

(i) Express b and c in terms of p and q.

(ii) Sketch the curve. Mark on your sketch the point of inflection and shade the region R. Describe the symmetry of the curve.

(iii) Show that $m - n = (q-p)^3$.

(iv) Show that the area of R is $\frac{1}{2}(q-p)^4$.

3 2011 S2 Q1

(i) Sketch the curve $y = \sqrt{1-x} + \sqrt{3+x}$.

Use your sketch to show that only one real value of x satisfies

$$\sqrt{1-x} + \sqrt{3+x} = x + 1\,,$$

and give this value.

(ii) Determine graphically the number of real values of x that satisfy

$$2\sqrt{1-x} = \sqrt{3+x} + \sqrt{3-x}\ .$$

Solve this equation.

4 1999 S2 Q7

The curve C has equation

$$y = \frac{x}{\sqrt{x^2 - 2x + a}}\ ,$$

where the square root is positive. Show that, if $a > 1$, then C has exactly one stationary point.

Sketch C when **(i)** $a = 2$ and **(ii)** $a = 1$.