STEP Support Programme

STEP 2 Probability and Statistics Questions: Hints

1 A very useful fact is that $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$.

We know that the sum of all the probabilities must be equal to 1, so we have $\displaystyle\sum_{k=1}^{\infty} A\frac{\lambda^k e^{-\lambda}}{k!} = 1$.

It will probably be helpful to take the constants outside of the sum (and remember that since λ is a constant then $e^{-\lambda}$ is a constant). The bit left inside the sum is similar to (but not equal to) e^{λ}.

When a question asks for the mean, it means the expectation $E(X)$. It will probably be easier to leave "A" as "A" whilst finding the mean and variance. You should be able to find some e^{λ} terms after a bit of manipulation (e.g. factorising out λ).

Remember that $\text{Var}(X) > 0$ and that $e^{-\lambda} < 1$.

X is not a Poisson distribution, but you can show that it is very close to one.

2 **(i)** The function is continuous, so the sections must meet up. In particular it must meet the x axis at $x = 1$ and $x = 4k$.

(ii) Since the ends of the sections meet up, you can use the two definitions of $f(x)$ at $x = 2k$ and $x = 4k$ to find two equations involving a, b and k which can be used to express a and b in terms of k (the equations for $x = 1$ and $x = k$ must also hold, but these are not very illuminating).

To find k note that the total area under $f(x)$ is equal to 1. To integrate $\ln(x)$ write it as $1 \times \ln(x)$ and use integration by parts.

(iii) The first thing you need to work out is which section the median lies in.

3 The first thing to do is read the question very carefully. Note that at the start of **each** game Xavier has a probability of p of winning the first point, i.e. it does not depend on the last point of the previous game.

(i) This part is much longer than the other two and most of the work is done here. If Younis wins, he might have won on the first game, or the first was drawn and he won on the second, or the first two were drawn and he won the third etc. so you need to consider an infinite sum. The probability that Younis wins a particular game is the same for every game and needs to be found (as does the probability that a game is drawn).

When trying to show $A > B$ it can sometimes be easier to show that $A - B > 0$ instead. For the last part differentiation is helpful.

(ii) If the game is fair, then the expected gain of both Younis and Xavier is 0.

(iii) Let $p = 0$, and then work out who wins each point in a game.

4 If X is the number of supermarkets in a circle of radius y then $X \sim \text{Po}(k\pi y^2)$. You need to find $\text{P}(X = 0)$.

$Y < y$ means that there is at least one supermarket within a circle of radius y centred on the chosen point.

$\text{P}(Y < y) = \int_0^y \text{f}(t)\text{d}t$, so to find the probability density function you need to differentiate $\text{P}(Y < y)$ with respect to y.

For the expectation and variance you need to use the definitions and evaluate the relevant integrals. The first sentence of the question gives a useful result and you will need integration by parts.

5 If George waits between 1 and 2 hours for his first text then he receives none in the first hour followed by **at least one** in the second hour. The given equation is a quadratic equation in e^{λ}. If we are to have $\lambda > 0$ then we need $\text{e}^{\lambda} > 1$.

In the case of Mildred we have $pe^{2\lambda} - \text{e}^{\lambda} + 1 = 0$ for each phone, but the two values of λ_1 and λ_2 are different. This means you can find the two values. It might be easier to find $\text{e}^{\lambda_1 + \lambda_2}$ before finding $\lambda_1 + \lambda_2$.

For the last part the total number of calls that Mildred gets on the two phones combined will be a Poisson distribution with mean $\lambda_1 + \lambda_2$.

6 Don't forget the request in the "stem". A sketch of $f(x)$ with a bit of explanation should be enough.

(i) It is probably easier to find $E(X)$ in terms of k and then substitute for k (which you can find from your sketch — remember that the total area under the graph is 1).

(ii) There are two different answers depending on whether M is greater than or less than k. The different cases occur when $ak \leqslant \frac{1}{2}$ and $ak \geqslant \frac{1}{2}$. Your sketch will probably be helpful.

(iii) There are two different cases. In each case it might be easier to show $M - E(X) < 0$.