STEP Support Programme

STEP 2 Trigonometry Questions: Hints

1 Start by using $\cos(3x) = \cos(2x + x)$. Do something very similar for $\sin(3x)$.

(i) Rewrite $\sin^3 x$ in terms of $\sin x$ and $\sin 3x$. There is one value of α which must give $\displaystyle\int_0^\alpha \mathrm{f}(x)\,\mathrm{d}x = 0$ from which you can find the value of $c = \cos(\alpha)$. The question is a "write down" which implies that not much work is needed.

(ii) Eustace believes the given statement for $n = 1$, so he will even get $\int \sin x\,\mathrm{d}x$ wrong. You will want to write Eustace's attempt in terms of c. There is one obvious value of α (and therefore c) for which Eustace will get the correct value, which can be used to help find the other values.

You are asked to find **all** the values, of which there are infinitely many! Refer to the "general solutions" part of the topic notes.

2 There are many ways to prove the result in the "stem". You could start on the left hand side or the right hand side.

(i) First find the value of x which will give $\frac{1}{4}\pi - \frac{1}{2}x = \frac{1}{8}\pi$. Then you can write $\frac{11}{24}\pi$ in terms of $\frac{1}{8}\pi$ and another useful fraction of π.

(ii) Since you are given both sides you can instead show an equivalent statement to be true. Remember to show all your working!

(iii) It looks like the previous work should be useful! Try using $x = \frac{11}{24}\pi$ in $(*)$.

3 Note that $\arctan x = \tan^{-1} x$. This is an integration by substitution question, and you will need to use a relationship between $\tan\theta$ and $\sec\theta$.

(i) **(a)** Try to find a substitution which will convert I into something of the same form as the "stem" result. You will find that $a = 1$.

(b) Quite a lot of manipulation of trigonometric functions is needed here, but you do know what you are aiming for. Completing the square might be a useful technique to consider. Remember that $\sin 2A = 2\sin A\cos A$.

(ii) This is very similar to part **(i)(b)**. Start with the same substitution as in the previous part and then use a second one to write the integral in the same form as in the stem.

4 This is quite a long question with lots of things to think about.

(i) The first thing that springs to mind is using $\sin 4\theta = 2\sin 2\theta\cos 2\theta$, but this won't help find the actual values of θ. You will need this technique later in this part to find the value of $\sin 18°$.

Start by using $\cos\alpha = \sin(90° - \alpha)$ or $\sin\alpha = \cos(90° - \alpha)$. Remember that if $\cos\alpha = \cos\beta$ it does not mean that $\alpha = \beta$, you could have $\alpha = -\beta$ etc.

(ii) Start by writing the equation in terms of $s = \sin x$ which will be a quadratic in s^2. The previous part might be useful.

(iii) "Hence" means that you need to use some of the previous parts. Try to find a value of 5α for which the equation had the same form as in part **(ii)**.