STEP Support Programme

STEP II Trigonometry Topic Notes

Radians —Most of the time, STEP questions will use radians (if they want you to use degrees then this will be made fairly obvious). It is helpful for you to know all the special angles etc. in terms of radians, but if you must convert them into degrees and back again then this is OK[1].

If you are in **radians** then we have:

$$\text{arc length:} \quad s = r\theta \quad \text{and area of a segment:} \quad A = \tfrac{1}{2}r^2\theta\,.$$

Special angles — these can be found by drawing an equilateral triangle with side length 1, and an isosceles right-angled triangle with equal sides length 1.

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Symmetry —The trigonometric functions have a lot of symmetry which can often be used to help solve equations etc.

$$\begin{array}{ccc} \cos(-\alpha) = \cos\alpha & \cos(\alpha + 2\pi) = \cos(\alpha) & \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha \\ \sin(\pi - \alpha) = \sin\alpha & \sin(\alpha + 2\pi) = \sin(\alpha) & \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha \\ \tan(\pi + \alpha) = \tan(\alpha) & \text{etc.} & \end{array}$$

There are more symmetries (such as $\sin(-\alpha) = -\sin\alpha$). You could memorise them all, or know that things like this exist and use sketches to derive them. The links connecting sin and cos are particular worth knowing about.

General solutions — STEP expects a little more here than A-level does. These results build upon the symmetry of the graphs.

If $\theta = \alpha$ is a solution of the equation $\cos\theta = k$, then so are $\theta = \alpha + 2\pi n$ and $\theta = -\alpha + 2\pi n$ (where n is an integer).

Similarly:

$$\text{If} \quad \sin\theta = \sin\alpha \quad \text{then} \quad \theta = \alpha + 2\pi n \quad \text{or} \quad \theta = (\pi - \alpha) + 2\pi n$$

$$\text{If} \quad \tan\theta = \tan\alpha \quad \text{then} \quad \theta = \alpha + \pi n$$

[1] Make sure that if calculus is involved that everything is in radians throughout!

Identities (not an exhaustive list!)

$$
\begin{aligned}
\tan\theta &\equiv \frac{\sin\theta}{\cos\theta}\\
\cos^2\theta + \sin^2\theta &\equiv 1\\
1+\tan^2\theta &\equiv \sec^2\theta\\
\cos(A\pm B) &\equiv \cos A\cos B \mp \sin A\sin B\\
\sin(A\pm B) &\equiv \sin A\cos B \pm \sin B\cos A\\
\tan(A\pm B) &\equiv \frac{\tan A \pm \tan B}{1\mp \tan A\tan B}\\
\sin(2A) &\equiv 2\sin A\cos A\\
\cos(2A) &\equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A\\
\tan(2A) &\equiv \frac{2\tan A}{1-\tan^2 A}
\end{aligned}
$$

Note that lots of these can be derived from others in the list, for example the third identity is the second one divided throughout by $\cos^2\theta$.

The compound angle formulae can be used to write a sum of $\cos x$ and $\sin x$ as a single trig function, e.g. $a\sin x + b\cos x = R\sin(x+\alpha)$ where $R=\sqrt{a^2+b^2}$ and $\tan\alpha = \frac{b}{a}$. This comes from:

$$
\begin{aligned}
R\sin(x+\alpha) &\equiv R\left(\sin x\cos\alpha + \cos x\sin\alpha\right)\\
&\equiv R\cos\alpha\sin x + R\sin\alpha\cos x\\
&\equiv a\sin x + b\cos x
\end{aligned}
$$

Equating coefficients then gives us $a = R\cos\alpha$ and $b = R\sin\alpha$. Squaring and adding gives $R^2 = a^2+b^2$ and dividing one by the other gives $\tan\alpha = \frac{b}{a}$. This technique can be used to solve $a\sin x + b\cos x = c$, and you can use a similar argument to write $a\sin x + b\cos x$ in the form $R\cos(x+\alpha)$.

Inverse trigonometric functions

If $y = \sin^{-1}x = \arcsin x$ and $-1\leqslant x\leqslant 1$ then $-\frac{\pi}{2} < y \leqslant \frac{\pi}{2}$ (if $|x|>1$ then y is not real).

If $y = \cos^{-1}x = \arccos x$ and $-1\leqslant x\leqslant 1$ then $0 < y \leqslant \pi$ (if $|x|>1$ then y is not real).

If $y = \tan^{-1}x = \arctan x$ and $x\in\mathbb{R}$, then $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Tangent half-angle substitution See this page for some background info.

If we let $\tan\frac{1}{2}A = t$ then we have:

$$\sin A = \frac{2t}{1+t^2} \quad \text{and} \quad \cos A = \frac{1-t^2}{1+t^2}$$