STEP Support Programme

STEP 3 Vectors Hints

1 **SPECIMEN S2 Q9**

(i) Start by drawing a diagram. You can also find the vector equation of the perpendicular line joining the centre of the sphere to the plane. This can be used to find the perpendicular distance between the plane and the centre of the sphere.

(ii) Start by considering $\dfrac{(\mathbf{x}\cdot\mathbf{c})\mathbf{c}}{|\mathbf{c}|^2}$, and by using $\mathbf{x}\cdot\mathbf{c} = |\mathbf{x}||\mathbf{c}|\cos\alpha$. Draw another diagram!

Draw a diagram showing $\mathbf{x}$ and $\mathbf{x}'$ and use this to describe geometrically how $\mathbf{x}'$ is related to $\mathbf{x}$.

2 **92 S2 Q9**

Start by rewriting $\mathbf{r}$ — gather together the λ and μ terms.

The equation $\mathbf{r}\cdot\mathbf{n} = p$ describes a plane — use your rewritten $\mathbf{r}$ to find a suitable $\mathbf{n}$ and then use a point in $\mathbf{r}$ to find p.

Remember that $\mathbf{a}\times\mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, and that the scalar product of two perpendicular vectors is equal to 0.

You can substitute some values of λ and μ.

3 93 S2 Q4

You can consider two parallel planes, both described by the direction vectors $\mathbf{m}_1$ and $\mathbf{m}_2$, one of which contains $\mathbf{p}_1$ (and hence the whole first line) and one that contains $\mathbf{p}_2$ (and hence the second line). The shortest (perpendicular) distance between the two lines is the same as the distance between these two planes.

Draw a sketch showing this and also the vector $\mathbf{p}_1 - \mathbf{p}_2$. Use a right-angled triangle to find an expression for the distance between the planes.

(i) Plug in the numbers into the formula given in the stem.

Remember that you can still get the marks for this part even if you have not done the stem of the question. In an exam when you only have a few minutes left and have not started a sixth question doing this part to hopefully gain another few marks is a reasonable strategy.

(ii) In this part the parameter is the same for both lines (as it is a position at a certain time). Find an expression for the vector between the two plane at time t and hence find an expression for the distance squared between them. This will be a quadratic in t, so can be minimised.

(iii) Imagine the paths of the two planes (a bit like two contrails). The pilot of A_2 can pick his planes speed so that at the time when A_1 is at the point on its path when the two paths are closest then (at the same time) A_2 is also at the point on its path when the two paths are closest.

4 95 S3 Q8

Start by finding the equation of the line perpendicular to plane π and which passes through the point X. Then solve simultaneous equations to find where the line and plane meet. This can be used to find the perpendicular distance of X from plane π.

For the next part, start by drawing a picture (perhaps just the sphere and C_1 to begin with). Find an equation for the line joining $\mathbf{r}_1$ to the centre of the sphere (which could have position vector $\mathbf{x}$).

Then draw a picture showing both circles. Remember that both of the circles are *unit circles* so they have the same radius.

Once you have the relationship $\mathbf{r}_1 + \lambda\mathbf{n}_1 = \mathbf{r}_2 \pm \lambda\mathbf{n}_2$ then you can manipulate this to show the other relationships.

To interpret the last relationship you might like to return to the first request of this question.