STEP Support Programme

STEP 2 Vectors Questions: Hints

1 Start by writing $m_{3/4}$ as a general 3D vector. Then use the dot product, for example $m_1 \cdot m_3 = |m_1||m_3| \cos\left(\frac{\pi}{4}\right)$. You are expecting two solutions (one being m_3 and one being m_4).

Remember that, for example, the direction $1\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ is the same as the direction $4\mathbf{i}+8\mathbf{j}+12\mathbf{k}$ (or any other multiple). You may like to pick a (non-zero) value of one component (but just check that this component cannot be zero first).

For parts **(i)** and **(ii)** it is helpful to find A, B, P and Q first.

(i) If vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular then $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$.

(ii) Try and find where they intersect and then show that this solution is not possible.

2 Start by drawing a clear diagram! Try also to work out what is happening geometrically, what do you know about OB and OC? How does OA relate to these? How does BC relate to OA?

The point D is found in a very similar way to point C, so you can use your previous work to help here without retracing your steps entirely. You need to find μ in terms of $\mathbf{a}$ and $\mathbf{b}$ (not least because it makes things easier later on!).

If three points are collinear, then the vector between any two of them is a multiple between the vector between one of these and the third one.

3 Start by drawing a clear diagram showing points A, B and C, making sure that the three points do not all lie on the same straight line.

It is very useful to note that the equation of the straight line through X and Y has the form $\mathbf{r} = t\mathbf{x} + (1-t)\mathbf{y}$ and the value of t determines where on the line XY a particular point is (it might not be in-between X and Y). You could try substituting some values of λ and μ in order to get a feel for what is happening (make sure your values are in the given range).

You may also find it helpful to rewrite the expressions for $\mathbf{p}$ and $\mathbf{q}$, such as $\mathbf{p} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b})$.

You will need to find an equation for the line PQ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ (start by finding it in terms of $\mathbf{p}$ and $\mathbf{q}$).

For the last bit, rewrite $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ so that there are two position vectors on each side.

4 The "Stem" needs an application of the scalar product.

(i) If L_1 makes the same angle with OA and OB you can use the scalar product to find an equation involving m, n and p. There are lots of possible lines making the same angle with OA and OB as we are in three dimensions.

If L_1 is the angle bisector of $\angle AOB$ then you know what angle L_1 makes with OA.

(ii) This is very similar to the previous part, and you might be able to use some of the working again (with different letters). This time we know what angle L_2 makes with OA, but there is no restriction on the angle it makes with OB.