STEP Support Programme

STEP 2 Miscellaneous Topic Notes

Factor theorem: $ax+b$ is a factor of the polynomial $\mathrm{f}(x)$ if and only if $\mathrm{f}\left(-\frac{b}{a}\right)=0$.

Remainder Theorem: The remainder when you divide the polynomial $\mathrm{f}(x)$ by $(ax+b)$ is $\mathrm{f}\left(-\frac{b}{a}\right)$.

Expansions

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots \binom{n}{n-1}ab^{n-1} + b^n$

Where $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ and n is a positive integer.

$(1+x)^k = 1 + kx + \dfrac{k(k-1)}{2!}x^2 + \dfrac{k(k-1)(k-2)}{3!}x^3 + \cdots$ where k is a rational number and $|x|<1$.

Arithmetic series
Recurrence relation: $t_n = t_{n-1} + d$
n^{th} term: $t_n = a + (n-1)d$
Sum of n terms: $S_n = \frac{1}{2}n\big(a+l\big) = \frac{1}{2}n\big(2a+(n-1)d\big)$
Sum of natural numbers: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Geometric series
Recurrence relation: $t_n = r \times t_{n-1}$
n^{th} term: $t_n = a \times r^{n-1}$
Sum of n terms: $S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{a(1-r^n)}{1-r}$
Infinite sum: $S_\infty = \dfrac{a}{1-r}$ for $|r|<1$

Exponential series
$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \cdots$

Coordinate geometry

Gradient between (x_1,y_1) and (x_2,y_2): $m = \dfrac{y_2-y_1}{x_2-x_1}$

Distance between (x_1,y_1) and (x_2,y_2): $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint of (x_1,y_1) and (x_2,y_2): $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Equation of a straight line with gradient m passing through (x_1,y_1): $y-y_1 = m(x-x_1)$

Equation of a circle radius R with centre (a,b): $(x-a)^2+(y-b)^2 = R^2$